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Realizations of perturbations of an observable pair with prescribed indices ☆

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ABSTRACT

It is well known that, when a full rank observable pair (C, A) is slightly perturbed, the new observability indices k' are majorized by the initial ones k, $k \succ k'$. Conversely, any indices k' majorized by k can be obtained by perturbing (C, A). The aim of this paper is the explicit construction of perturbations of (C, A) which have the desired indices k' by means of a sequence of uniparametrical versal perturbations. Even more, using versal deformations we refine this construction in such a way that the perturbation has the maximum possible number of zeros and no parameters in the square part.

1. Introduction

It is well known [4,2] that, when a full rank observable pair $(C, A) \in \mathbb{C}^{m\times n} \times \mathbb{C}^{n\times n}$, with observability indices $k = (k_1, k_2, \ldots, k_m)$ in non-increasing order, is slightly perturbed, the new observability

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indices $k' = (k'_1, k'_2, \ldots, k'_m)$ in non-increasing order are *majorized* [5] by the initial ones k, that is to say:

$$k_1 \geqslant k_1', k_1 + k_2 \geqslant k_1' + k_2', \ldots, \sum_{j=1}^{m} k_j = \sum_{j=1}^{m} k_j'.$$

Conversely, any indices k' majorized by k can be obtained by perturbing (C, A).

The aim of this paper is the explicit construction of "minimal"perturbations of (C, A) having the desired indices k', where "minimal"means that one preserves as many entries of (C, A) as possible. We can guarantee this "minimality" because the only allowed perturbations of (C, A) are those in the miniversal deformation in Theorem 2.4 [1]: The number of parameters is minimal according to Arnold's theory and each one perturbs a unique entry of (C, A).

The construction is attempted in two steps. First (Theorem 3.7), we obtain a realization of k', that is to say, an explicit perturbation of (C, A) having the desired indices k'. Next (Theorem 4.11), we move the parameters appearing there in order to place them in the entries in the miniversal deformation in Theorem 2.4 [1]. Again Arnold's theory ensures that this replacement is possible, but the explicit construction is not trivial.

In Section 2 we establish the notation, we recall what we understand by BK-canonical form of a pair (Definition 2.2), what a miniversal deformation is and Theorem 2.4, which gives the miniversal deformation obtained in [1].

Finally, we define what we mean by elementary versal perturbation, realization and versal realization of any tuple of indices.

Section 3 is devoted to obtaining realizations of tuples of indices majorized by the given one, by means of a (good) sequence of the so-called elementary ones.

First, we study the elementary versal perturbations. We see that the new observability indices k' differ from the initial ones in only two of them: $k' = (\ldots, k_i - p, \ldots, k_j + p, \ldots)$. We say that k' is an elementary change of k.

For any elementary versal perturbation, one computes explicitly the change of basis that reduces this versal perturbation to its BK-canonical form.

For a general tuple of indices k^f majorized by k, we consider a sequence of m-tuples of indices

$$k^{(0)} = k, k^{(1)}, k^{(2)}, \ldots, k^{(l)} = k^f$$

of elementary changes, that is to say: each $k^{(j)}$ is majorized by the previous one $k^{(j-1)}$ and both differ only in two indices. Thus, each $k^{(j)}$ can be realized as an elementary versal perturbation of the previous one. In order to simplify the computations, we restrict ourselves to sequences of this kind having minimal length, which we call good sequences.

Thus, given a full rank observable pair (C, A), with observability indices $k = (k_1, k_2, \ldots, k_m)$ and a tuple of indices k^f majorized by k, we construct a realization of k^f by means of elementary versal perturbations in a good sequence (Theorem 3.7).

We notice through an example that this realization is not good in the sense that the parameters are located in both matrices.

In order to correct it, the remainder Section 4 is devoted to changing the intermediate perturbations in order to obtain a final realization with all the parameters in the entries of the initial miniversal deformation (Theorem 4.11 and Corollary 4.12).

2. Preliminaries

Let $\mathcal{M} = \{(C, A) : A \in \mathbb{C}^{n\times n}, C \in \mathbb{C}^{m\times n}\}$ be the differentiable manifold of pairs of matrices and let $\mathcal{M}^*$ be the open dense subset of $\mathcal{M}$ formed by the observable pairs with rank $C = m$, that is to say, the *full rank observable pairs*.