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Signal decomposition and analysis via extraction of frequencies

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ABSTRACT

Time–frequency analysis is central to signal processing, with standard adaptation to higher dimensions for imaging applications, and beyond. However, although the theory, methods, and algorithms for stationary signals are well developed, mathematical analysis of non-stationary signals is almost nonexistent. For a real-valued signal defined on the time-domain $\mathbb{R}$, a classical approach to compute its instantaneous frequency (IF) is to consider the amplitude–frequency modulated (AM–FM) formulation of its complex (or analytic) signal extension, via the Hilbert transform. In a popular paper by Huang et al., the so-called empirical mode decomposition (EMD) scheme is introduced to separate such a signal as a sum of finitely many intrinsic mode functions (IMFs), with a slowly oscillating signal as the remainder, so that more than one IFs of the given signal can be computed by extending each IMF to an AM–FM signal component. Based on the continuous wavelet transform (CWT), the notion of synchrosqueezing transform (SST), introduced by Daubechies and Maes in 1996, and further developed by Daubechies, Lu, and Wu (DLW) in a 2011 paper, provides another approach to extract more than one IFs of the signal on $\mathbb{R}$. Furthermore, by introducing a list of fairly restrictive conditions on the adaptive harmonic model (AHM), the DLW paper also derives a theory for estimating the signal components according to this model, by using the IFs with estimates from the SST.

The objective of our present paper is to introduce another mathematical theory, along with rigorous methods and computational schemes, to achieve a more ambitious goal than the SST approach, first to extract the polynomial-like trend from the source signal, then to compute the exact number of signal components according to a less restrictive AHM model, then to obtain better estimates of the IFs and instantaneous amplitudes (IAs) of the signal components, and finally to separate the signal components from the (blind) source signal. Furthermore, our computational scheme can be realized in near-real-time, and our mathematical theory has direct extension to the multivariate setting.

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1. Introduction

Time–frequency analysis is central to practically all areas of “signal processing”; and extensions of time–frequency representations, methods, and algorithms to higher dimensions are also used in most applications on imagery analysis and processing. In addition, time–frequency representation in terms of (discrete) cosine polynomials is adopted by audio, image, and video industry standards for (lossy) data compression, including: MP3 audio, JPEG image, as well as MPEG and H.264 videos. When a signal (or function) is represented by a cosine polynomial, such as

$$G(t) = a_0 + \sum_{k=1}^{K} a_k \cos(2\pi k t), \tag{1.1}$$

it is clear that G is a superposition of the signal components $f_k(t) = a_k \cos(2\pi kt)$, for $k = 1, \dots, K$, where the frequency of the component f_k is $\omega_k = k$ Hz. Such signals are called stationary signals, meaning that the frequencies $\omega_k = k$ of $G(t)$, with $k = 1, \dots, K$, do not change with the time variable t, where K may be considered as (an upper bound of) the bandwidth of the signal. However, real-world signals are mainly non-stationary, meaning that their frequencies may change with time. Unfortunately, while the mathematical theory of stationary signals is well developed, mathematical analysis of non-stationary signals is almost nonexistent. Although the concept of “time–varying frequencies” was already disclosed in the Bell System Technical Report [3], the pioneering work on non-stationary signal analysis is often attributed to the landmark paper [17] of Dennis Gabor, along with the paper [26] of Van der Pol, published in the same volume of J.IEE in 1946. In [17], for a given real-valued signal $f(t)$ defined for $t \in \mathbb{R}$, Gabor introduced the notion of the complex signal extension $G^c = G + i\mathcal{H}G$ (also called the “analytic signal extension” of f in the current signal processing literature), by applying the Hilbert transform $\mathcal{H}$ to G over $\mathbb{R}$. Hence, by writing the analytic signal G^c in its polar form, namely: $G^c(t) = A(t)\exp(i2\pi\phi(t))$ (called amplitude–frequency modulated (AM–FM) signal representation in the current signal processing literature), with $A(t) \geq 0$, the given signal is represented by $G(t) = A(t)\cos(2\pi\phi(t))$, by taking the real part of the polar form of $G^c(t)$. Consequently, it is natural to define the “instantaneous frequency” (IF) of the given signal by the derivative $\phi'(t)$ of the phase function $\phi(t)$. There have been other attempts to define instantaneous frequencies, particularly in the early 1990s, with perhaps the most popular ones based on the Wigner–Ville distribution method (see, in particular, [1,2]). However, all such definitions suffer the same defect as the above discussion, in that the definition applies to one and only one frequency value, $\phi'(t)$, of the given signal $G(t)$ at the desired time instant $t \in \mathbb{R}$. This is certainly unacceptable, since any of such definitions does not allow time–frequency analysis of just about all signals. After all, only the simple harmonic signal $G(t) = A(t)\cos(2\pi\phi(t))$ has one IF for any $t \in \mathbb{R}$.

To allow for more than one frequencies, as in the stationary setting (1.1) with $K > 1$, N. Huang et al. [19,18] introduce a signal decomposition procedure, called the “empirical mode decomposition” (EMD) scheme, to decompose the given (non-stationary) signal G into a finite sum of “intrinsic mode functions” (IMFs) f_j, with a slowly oscillating function T as the remainder, and apply the Hilbert transform $\mathcal{H}$, as in [17], to extend each IMF f_j to an AM–FM signal. In other words, for a given real-valued (stationary or non-stationary) signal $G(t)$, by taking the real part of the Hilbert spectrum (that is, the sum of the AM–FM analytic signal extensions of f_js), the EMD of $G(t)$ can be formulated as

$$G(t) = f(t) + T(t), \tag{1.2}$$

with