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New ultimate bound sets and exponential finite-time synchronization for the complex Lorenz system

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In this paper, by using the optimization idea, a new ultimate bound for the complex Lorenz system is derived. It is shown that a hyperelliptic estimate of the ultimate bound set can be analytically calculated based on the optimization method and the Lagrange multiplier method. And based on the ellipsoidal bound set and set operations, one further obtains a more conservative boundary for each variable in the complex system, which only relies on the system parameters. Afterwards, the estimated results are applied to the exponential finite-time synchronization of the complex Lorenz system. Especially, the designed control depends on the parameters of the exponential convergence rate, the finite-time convergence rate, the bound of the initial states of the master system, and the system parameter. Finally, numerical simulations are given to verify the effectiveness and correctness of the obtained results.

1. Introduction

Chaotic and hyperchaotic systems appear in several important applications in many physical and biological systems, as well as secure communications using chaotic and hyperchaotic signals [2,22,11].

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Recently, many chaotic complex systems have been proposed and studied since Fowler et al. introduced the complex Lorenz equations [7]. For instance, Mahmoud et al. introduced the complex Chen and complex Lü systems and showed their chaotic attractors and the stability properties of their fixed points [19]. By adding a state feedback controller and using complex periodic forcing, a new hyperchaotic complex Lü system [21] was constructed. In [31], the authors investigated robust synchronization of complex switched networks with parametric uncertainties and two types of delays. Wang et al. analyzed global synchronization of complex dynamical networks with network failures [36]. Also, the synchronization of uncertain complex dynamical networks via adaptive control is investigated in [32].

In particular, the estimate of the ultimate bound for a chaotic system is of great importance for chaos control, chaos synchronization, Hausdorff dimension and the Lyapunov dimension of chaotic attractors [25,18,9,38,37]. If we can show that a chaotic or a hyperchaotic system has a globally attractive set, then we know that the system cannot possess equilibrium points, periodic or quasi-periodic solutions, or other chaotic or hyperchaotic attractors outside the globally attractive set. Ever since the Lorenz system was put forward, its ultimate bound has been investigated by G.A. Leonov et al. [15]. There are a few studies which have discussed the solution bounds and invariant sets of the chaotic systems, such as the Lorenz [13,14], the hyperchaotic Lorenz–Haken system [17], Lure systems [27], Lorenz–Stenflo system [34,26], the Lorenz family of chaotic systems [16], a new chaotic system [28], stochastic cellular neural networks with delays [30], one tumor growth model [29], perturbed time-delay systems [24], and a class of high dimensional quadratic autonomous chaotic systems [35] are investigated. If the initial condition is unknown, it is very difficult to predict the response trajectory of the chaotic system. Furthermore, it is some times difficult to obtain the exact solutions of a complex system but we can estimate a solution bound in order to analyze the system, or for reducing computation burdens or complexity of design controllers. Recently, based on the optimization method and the Lagrange multiplier method, the ultimate boundness of the general quadratic autonomous dynamical systems (GQADS) was reported by Wang et al. [37].

In the past decades, the control and synchronization of chaotic systems has attracted much attention and some relevant theoretical results have been established [12,3,8,1,6,5]. On the other hand, minimizing the synchronization time is essential for achieving fast communication synchrony; and this could be done by means of finite-time control. Different control methods, such as output feedback control and finite time observer were constructed to stabilize nonlinear systems in finite time [23,33]. It was demonstrated that the finite-time control techniques have better robustness and disturbance rejection properties [33,4].

In 2006, Mahmoud et al. studied basic dynamical properties and chaotic synchronization of complex Lorenz system [20]. This system arises in many important applications in physics, for example, in laser physics and rotating fluids dynamics. According to our knowledge, the explicit bound sets for the complex Lorenz system have not been studied yet. The meaningful contribution of our work is the calculation of an explicit ultimate bound set for a complex chaotic system. In this paper, to get the analytical expression of the ultimate boundary region, the key is to calculate the analytical solution of the maximum optimization problem. Furthermore, we derive a more conservative boundary for each state variable in the complex Lorenz system. Utilizing the bounds obtained, a controller is proposed to achieve the exponential finite-time synchronization. In fact, based on the finite-time stability, Lyapunov theory and the ultimate bound of the complex Lorenz systems, a controller is designed which consists of two parts: one achieves exponential synchronization, and the other realizes finite-time synchronization within a pre-specified convergence time t_1. It is obvious that the proposed control form is much more simply implemented.

The rest of this paper is organized as follows. In Section 2, we give some preliminary definitions. Section 3 introduces an approach for the ultimate bound estimation of the complex Lorenz system. In Section 4, new criteria are obtained to ensure the finite-time synchronization of complex Lorenz system. Conclusions are drawn in Section 5.

2. Some notations and preliminaries

In this section, we introduce an existing method of ultimate bound for general quadratic autonomous dynamical system [37].