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Asymptotic stability of wave patterns to compressible viscous and heat-conducting gases in the half-space

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Abstract

We study the large-time behavior of solutions to the compressible Navier–Stokes equations for a viscous and heat-conducting ideal polytropic gas in the one-dimensional half-space. A rarefaction wave and its superposition with a non-degenerate stationary solution are shown to be asymptotically stable for the outflow problem with large initial perturbation and general adiabatic exponent.

MSC: primary 35B35; secondary 35B40, 35Q35, 76N10

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1. Introduction

The one-dimensional motion of a compressible viscous and heat-conducting gas in the half-space $\mathbb{R}_+ := (0, \infty)$ can be formulated by the compressible Navier–Stokes equations

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + P)_x = (\mu u_x)_x, \\ (\rho E)_t + (\rho u E + uP)_x = (\kappa\theta_x + \mu u u_x)_x, \end{cases} \tag{1.1}$$

where $t > 0$ and $x \in \mathbb{R}_+$ stand for the time variable and the spatial variable, respectively, and the primary dependent variables are the density ρ, the velocity u and the temperature θ. The specific total energy $E = e + \frac{1}{2}u^2$ with e being the specific internal energy. It is known from thermodynamics that only two of the thermodynamic variables ρ, θ, P (pressure), e and s (specific entropy) are independent. We focus on the ideal polytropic gas, which is expressed in normalized units by the following constitutive relations

$$P = R\rho\theta, \quad e = c_v\theta, \quad s = c_v \ln(\rho^{1-\gamma}\theta), \tag{1.2}$$

where $R > 0$ is the gas constant, $\gamma > 1$ the adiabatic exponent and $c_v = R/(\gamma - 1)$ the specific heat at constant volume. Positive constants μ and κ are the viscosity and the heat conductivity, respectively.

The system (1.1)–(1.2) is supplemented with the initial condition

$$(\rho, u, \theta)|_{t=0} = (\rho_0, u_0, \theta_0), \tag{1.3}$$

which is assumed to satisfy the far-field condition

$$\lim_{x\to\infty} (\rho_0, u_0, \theta_0)(x) = (\rho_+, u_+, \theta_+), \tag{1.4}$$

where $\rho_+ > 0$, u_+ and $\theta_+ > 0$ are constants. For boundary conditions, we take

$$(u, \theta)(t, 0) = (u_-, \theta_-), \tag{1.5}$$

where u_- and $\theta_- > 0$ are constants. The initial data (1.3) is assumed to satisfy certain compatibility conditions as usual.

The boundary condition $u(t, 0) = u_- < 0$ means that the fluid blows out from the boundary, and hence the initial boundary value problem (1.1)–(1.5) with $u_- < 0$ is called the outflow problem. The problem (1.1)–(1.5) with $u_- = 0$ is called the impermeable wall problem, which has been studied in [6,7,20,21,31] and so on. According to the theory of well-posedness for initial boundary value problem, one has to impose one extra boundary condition $\rho(t, 0) = \rho_-$ on $\{x = 0\}$ for the case when $u_- > 0$. This case is called the inflow problem and has been investigated by Matsumura et al. [4,6,9,22,27,28]. We refer to Matsumura [19] for a complete classification about the large-time behaviors of solutions to initial boundary value problems of the isentropic compressible Navier–Stokes equations in the half-space $\mathbb{R}_+$.

The main purpose of this article is to study the large-time behavior of solutions to the outflow problem (1.1)–(1.5). The nonlinear stability of the stationary solution, the rarefaction wave and their composition has been addressed in [15,26] under small initial perturbation. For large