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Remarks on the well-posedness of Camassa–Holm type equations in Besov spaces

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Abstract

In this paper, we prove the solution map of the Cauchy problem of Camassa–Holm type equations depends continuously on the initial data in nonhomogeneous Besov spaces in the sense of Hadamard by using the Littlewood–Paley theory and the method introduced by Kato [37] and Danchin [21].

Keywords: Camassa–Holm type equations; Littlewood–Paley theory; The continuity of the solution map; Nonhomogeneous Besov spaces

1. Introduction

In recent twenty years, Camassa–Holm type equations have attracted much attention as a class of integrable shallow water wave equations with peakons.

The first well-known integrable member of Camassa–Holm type equations is the Camassa–Holm (CH) equation [5]:

$$(1-\partial_x^2)u_t = -(3uu_x - 2u_xu_{xx} - uu_{xxx}).$$

The CH equation can be regarded as a shallow water wave equation [5,15]. It is completely integrable [5,7], has a bi-Hamiltonian structure [6,28], and admits exact peaked solitons of the

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form $ce^{-|x-ct|}$, $c>0$, which are orbitally stable [17]. It is worth mentioning that the peaked solitons present the characteristic for the traveling water waves of greatest height and largest amplitude and arise as solutions to the free-boundary problem for incompressible Euler equations over a flat bed, cf. [9,13,14,46].

The local well-posedness for the Cauchy problem of the CH equation in Sobolev spaces and Besov spaces was discussed in [10,11,19,45]. It was shown that there exist global strong solutions to the CH equation [8,10,11] and finite time blow-up strong solutions to the CH equation [8, 10–12]. The existence and uniqueness of global weak solutions to the CH equation were proved in [16,51]. The global conservative and dissipative solutions of CH equation were discussed in [3,4].

The second well-known integrable member of Camassa–Holm type equations is the Degasperis–Procesi (DP) equation [23]:

$$(1-\partial_x^2)u_t = -(4uu_x - 3u_x u_{xx} - uu_{xxx}).$$

The DP equation can be regarded as a model for nonlinear shallow water dynamics and its asymptotic accuracy is the same as for the CH shallow water equation [24]. The DP equation is integrable and has a bi-Hamiltonian structure [22]. An inverse scattering approach for computing n-peakon solutions to the DP equation was presented in [43]. Its traveling wave solutions was investigated in [38,47].

The local well-posedness of the Cauchy problem of the DP equation in Sobolev spaces and Besov spaces was studied in [32,35,54]. Similar to the CH equation, the DP equation has also global strong solutions [40,55,57] and finite time blow-up solutions [26,27,40,41,54–57]. On the other hand, it has global weak solutions [2,26,56,57].

Although the DP equation is similar to the CH equation in several aspects, these two equations are truly different. One of the novel features of the DP different from the CH equation is that it has not only peakon solutions [22] and periodic peakon solutions [56], but also shock peakons [42] and the periodic shock waves [27].

The third well-known integrable member of Camassa–Holm type equations is the two-component Camassa–Holm shallow water system (2CH) [18]:

$$\begin{cases} m_t + um_x + 2u_x m + \sigma\rho\rho_x = 0, \\ \rho_t + (u\rho)_x = 0, \end{cases} \tag{1.1}$$

where $m = u - u_{xx}$ and $\sigma = \pm 1$. Local well-posedness for (2CH) with the initial data in Sobolev spaces and in Besov spaces was investigated in [18], [25], and [33], respectively. The blow-up phenomena and global existence of strong solutions to (2CH) in Sobolev spaces were obtained in [25,29,34]. The existence of global weak solutions for (2CH) with $\sigma = 1$ was investigated in [30]. The persistence property of (2CH) was studied in [31].

The fourth well-known integrable member of Camassa–Holm type equations is the Novikov equation [44]:

$$(1-\partial_x^2)u_t = 3uu_x u_{xx} + u^2 u_{xxx} - 4u^2 u_x.$$

The most difference between the Novikov equation and the CH and DP equations is that the former one has cubic nonlinearity and the latter ones have quadratic nonlinearity.