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Concentrating bounded states for a class of singularly perturbed Kirchhoff type equations with a general nonlinearity ☆

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Abstract

We are concerned with the following Kirchhoff type equation:

$$\begin{cases} -\left(\varepsilon^2 a + \varepsilon b \displaystyle\int_{\mathbb{R}^3} |\nabla u|^2\right)\Delta u + V(x)u = f(u) \text{ in } \mathbb{R}^3, \\ u > 0,\ u \in H^1(\mathbb{R}^3), \end{cases}$$

where ε is a small positive parameter, $a, b > 0$. Under general conditions on f due to Zhang et al. (2014) [37], we construct a family of positive solutions $u_\varepsilon \in H^1(\mathbb{R}^3)$ which concentrates around the local minima of V as $\varepsilon \to 0$. Our result completes the study made in Figueiredo et al. (2014) [10] in the sense that, only the subcritical growth was considered.

MSC: 35J20; 35J60; 35J92

Keywords: Existence; Concentration; Kirchhoff type equation; Critical growth

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1. Introduction and main result

We study the following Kirchhoff type equation:

$$\begin{cases} -\Big(\varepsilon^2 a + \varepsilon b \int\limits_{\mathbb{R}^3} |\nabla u|^2\Big)\Delta u + V(x)u = f(u) \text{ in } \mathbb{R}^3, \\ u > 0, \;\; u \in H^1(\mathbb{R}^3), \end{cases} \tag{1.1}$$

where ε is a small positive parameter, $a, b > 0$ and the potential $V : \mathbb{R}^3 \to \mathbb{R}$ is a continuous function satisfying:

(V_1) $\inf\limits_{x\in\mathbb{R}^3} V(x) = \alpha > 0$;
(V_2) There is a bounded domain Λ such that

$$V_0 := \inf_{\Lambda} V < \min_{\partial\Lambda} V$$

and we set $\mathcal{M} := \{x \in \Lambda;\ V(x) = V_0\}$. This kind of hypothesis was first introduced by M. del Pino and P.L. Felmer in [26]. The nonlinearity $f : \mathbb{R} \to \mathbb{R}$ is a continuous function. Since we are looking for positive solutions, we may assume that $f(s) = 0$ for $s < 0$. Furthermore, we need the following conditions:

(f_1) $\lim\limits_{t\to 0^+} f(t)/t = 0$;
(f_2) $\lim\limits_{t\to +\infty} f(t)/t^5 = \kappa > 0$;
(f_3) $\exists\lambda > 0$ and $2 < q < 6$ such that $f(t) \geq \lambda t^{q-1} + t^5$ for $t \geq 0$.

(f_1)–(f_3) were first introduced by J. Zhang, Z. Chen, W. Zou [37] and can be regarded as an extension of the celebrated Berestycki–Lions' type nonlinearity (see [4,5]) to the critical growth. And we note that $f(t) = \lambda t^{q-1} + t^5 (2 < q < 6)$ is a special case of (f_1)–(f_3).

Remark. Without loss of generality, in the present paper, we assume that $\kappa = 1$ and $0 \in \mathcal{M}$.

Problem (1.1) is a variant type of the following Dirichlet problem of Kirchhoff type

$$\begin{cases} -\Big(a + b \int\limits_{\Omega} |\nabla u|^2\Big)\Delta u = f(x, u) \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1.2}$$

where $\Omega \subset \mathbb{R}^3$ is a smooth domain. Such problems are often referred to be nonlocal because of the presence of the term $(\int_\Omega |\nabla u|^2)\Delta u$ which implies that the equation (1.2) is no longer a pointwise identity. This phenomenon provokes some mathematical difficulties, which make the study of such a class of problems particularly interesting. On the other hand, problem (1.2) is related to the stationary analogue of the equation