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Instability of modes in a partially hinged rectangular plate

Vanderley Ferreira Jr [a], Filippo Gazzola [b,*], Ederson Moreira dos Santos [a]

[a] *Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, Caixa Postal 668, CEP 13560-970, São Carlos, SP, Brazil*

[b] *Dipartimento di Matematica, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy*

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Abstract

We consider a thin and narrow rectangular plate where the two short edges are hinged whereas the two long edges are free. This plate aims to represent the deck of a bridge, either a footbridge or a suspension bridge. We study a nonlocal evolution equation modeling the deformation of the plate and we prove existence, uniqueness and asymptotic behavior for the solutions for all initial data in suitable functional spaces. Then we prove results on the stability/instability of *simple modes* motivated by a phenomenon which is visible in actual bridges and we complement these theorems with some numerical experiments.

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Keywords: Nonlocal plate equation; Well-posedness; Asymptotic behavior; Stability

Contents

* Corresponding author.

E-mail addresses: vanderley.cn@gmail.com (V. Ferreira), filippo.gazzola@polimi.it (F. Gazzola), ederson@icmc.usp.br (E. Moreira dos Santos).

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1. Introduction

We consider a thin and narrow rectangular plate where the two short edges are hinged whereas the two long edges are free. This plate aims to represent the deck of a bridge, either a footbridge or a suspension bridge. In absence of forces, the plate lies flat horizontally and is represented by the planar domain $\Omega = (0,\pi) \times (-l,l)$ with $0 < l \ll \pi$. The plate is subject to dead and live loads acting orthogonally on Ω: these loads can be either pedestrians, vehicles, or the vortex shedding due to the wind. The plate is also subject to edge loads, also called buckling loads, that are compressive forces along the edges: this means that the plate is subject to prestressing.

We follow the plate model suggested by Berger [6]; see also the previous beam model suggested by Woinowsky-Krieger [31] and, independently, by Burgreen [8]. Then, the nonlocal evolution equation modeling the deformation of the plate reads

$$\begin{cases} U_{tt} + \delta U_t + \Delta^2 U - \phi(U) U_{xx} = F & \text{in } \Omega \times (0,T) \\ U = U_{xx} = 0 & \text{on } \{0,\pi\} \times [-l,l] \\ U_{yy} + \sigma U_{xx} = U_{yyy} + (2-\sigma) U_{xxy} = 0 & \text{on } [0,\pi] \times \{-l,l\} \\ U(x,y,0) = U_0(x,y), \qquad U_t(x,y,0) = V_0(x,y) & \text{in } \Omega \end{cases} \tag{1}$$

where the nonlinear term ϕ is defined by

$$\phi(U) = -P + S \int_\Omega U_x^2 ,$$

and carries a nonlocal effect into the model. Here $S > 0$ depends on the elasticity of the material composing the deck, $S \int_\Omega U_x^2$ measures the geometric nonlinearity of the plate due to its stretching, and $P > 0$ is the prestressing constant: one has $P > 0$ if the plate is compressed and $P < 0$ if the plate is stretched. The constant σ is the Poisson ratio: for metals its value lies around 0.3 while for concrete it is between 0.1 and 0.2. We assume throughout this paper that

$$0 < \sigma < \frac{1}{2}. \tag{2}$$

The function $F : \Omega \times [0,T] \to \mathbb{R}$ represents the vertical load over the deck and may depend on time while δ is a damping parameter. Finally U_0 and V_0 are, respectively, the initial position and velocity of the deck. The boundary conditions on the short edges are named after Navier [24] and model the fact that the plate is hinged in connection with the ground; note that $U_{xx} = \Delta U$ on