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The motion of closed hypersurfaces in the central force fields

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Abstract

This paper studies the large time existence for the motion of closed hypersurfaces in a radially symmetric potential. Physically, this surface can be considered as an electrically charged membrane with a constant charge per area in a radially symmetric potential. The evolution of such surface has been investigated by Schnürer and Smoczyk [20]. To study its motion, we introduce a quasi-linear degenerate hyperbolic equation which describes the motion of the surfaces extrinsically. Our main results show the large time existence of such Cauchy problem and the stability with respect to small initial data. When the radially symmetric potential function $v \equiv 1$, the local existence and stability results have been obtained by Notz [18]. The proof is based on a new Nash–Moser iteration scheme.

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Keywords: Hyperbolic mean curvature flow; Quasi-linear wave equation; Smooth solution; Nash–Moser iteration; Stability

1. Introduction and main results

Let Σ be an oriented smooth closed manifold of dimension n, and $(\mathcal{M}, \tilde{g})$ be the Euclidean space, *i.e.* $\mathcal{M} = \mathbf{R}^{n+1}$ and $\tilde{g}$ be the Euclidean metric. Consider a smooth family of immersions $F : [0, T] \times \Sigma \longrightarrow \mathcal{M}$, we define an action integral of the form

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$$\mathcal{S}(F)=\int_0^T \mathcal{K}(F)-\mathcal{V}(F)dt,$$

where $\mathcal{K}$ is the kinetic energy and $\mathcal{V}=V+\mathcal{J}$ is the total inner energy, V is a radially symmetric potential energy and $\mathcal{J}$ is the inner pressure.

We fix a reference measure $d\mu$ on Σ with a smooth density function defining a mass distribution on Σ. Then the total kinetic energy of all the points of the surface is

$$\mathcal{K}(F)=\frac{1}{2}\int_\Sigma |\partial_t F|^2 d\mu.$$

We denote $d\mu_t$ as the induced surface measure of the induced metric $g=F(t)^*\tilde{g}$ at time t, φ denotes a smooth, radially symmetric function (reflecting the presence of a central force) depending on $s:=\frac{|F|^2}{2}$. The radially symmetric potential energy of the hypersurface $F(\Sigma)$ is defined by

$$V(F)=\int_\Sigma v(s(F))d\mu_t,$$

where v is defined by

$$v(s)=\exp(-\frac{n}{2}\int_1^s \frac{\eta(w)}{w}dw), \tag{1.1}$$

$$\varphi(s)=-\frac{\partial_w v(w)}{v(w)}=\frac{n}{2w}\eta(w),$$

where $\eta:\mathbf{R}^+\longrightarrow\mathbf{R}$ is a smooth function.

The inner pressure is defined as

$$\mathcal{J}(F)=-\rho\log(\frac{Vol(F)}{Vol_0}),$$

where $\rho>0$ denotes a parameter which determines strength of the influence of the inner pressure compared to the surface tension, $Vol(F)$ denotes the enclosed volume of the surface $F(\Sigma)$. The initial enclosed volume Vol_0 as well as the constant ρ are included for scaling reasons. This inner pressure is motivated by that of an ideal gas with constant temperature, i.e. proportional to $Vol^{-1}(F)$. One can see [17,18] for more details on the inner pressure.

Then the action integral is

$$\mathcal{S}(F)=\frac{1}{2}\int_0^T\int_\Sigma |\partial_t F|^2 d\mu dt-\int_0^T\int_\Sigma v(s(F))d\mu_t dt+\rho\int_0^T \log(\frac{Vol(F)}{Vol_0})dt.$$

The Euler–Lagrange equation of functional $\mathcal{S}(F)$ is