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The Łojasiewicz–Simon gradient inequality for open elastic curves

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Abstract

In this paper we consider the elastic energy for open curves in Euclidean space subject to clamped boundary conditions and obtain the Łojasiewicz–Simon gradient inequality for this energy functional. Thanks to this inequality we can prove that a (suitably reparametrized) solution to the associated L^2-gradient flow converges for large time to an elastica, that is to a critical point of the functional.
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1. Introduction

In the past years a considerable number of papers dealing with the long-time existence of motion of curves by the L^2-gradient flow for the elastic energy have appeared in the literature. Closed curves subject to a length/area constraint of some sort and with or without an inextensibility condition have been studied for instance in [8,27,32,14,18,25,26]; open curves subject to

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different types of boundary conditions and a constraint on the length have been treated in [11,17,9,10], curves of infinite length in [24].

However, in several of the above frameworks the question of asymptotical convergence to an equilibrium point of the gradient system has not been satisfactorily answered. Indeed, in many of the works just mentioned (see for instance [8,11,17,9,10]), the method of proof chosen to show long-time existence allows only to infer that for a sequence of time converging to infinity there exists a subsequence of suitably reparametrized curves that converge to a critical point of the energy functional. Thus, in principle different sequences could converge to different critical points. Motivation for this work is to show that this does not happen and that given an initial smooth curve the whole flow converges (after an appropriate reparametrization) to a stationary solution.

We give here a detailed proof for the setting presented in [17], that is for the elastic flow of open curves subject to a constraint on the growth of the length (obtained by adding a suitable penalty term in the energy functional) and clamped boundary conditions (i.e. the two boundary points of the curve and its tangents are kept fixed along the evolution).

More precisely, let us recall that the elastic energy for a regular and sufficiently smooth curve $f : I \to \mathbb{R}^d$, $f = f(x)$, is given by

$$\mathcal{E} : f \mapsto \frac{1}{2}\int_I |\vec{\kappa}|^2 \,\mathrm{d}s_f = \frac{1}{2}\int_I |\vec{\kappa}|^2 \,|f_x|\,\mathrm{d}x, \tag{1.1}$$

with $\vec{\kappa}$ the curvature vector, that is $\vec{\kappa} = \partial_{s_f}\partial_{s_f} f$ where $\partial_{s_f} = \frac{1}{|f_x|}\partial_x$. Here and in the following, $d \in \mathbb{N}$, $d \geq 2$, and $I := [0, 1] \subset \mathbb{R}$. It is well known that the energy $\mathcal{E}$ is a geometric functional, i.e. it is invariant under reparametrizations of the curve f, and that the L^2-gradient of the elastic energy is given by

$$\nabla_{L^2}\mathcal{E}(f) = \nabla_{s_f}^2\vec{\kappa} + \frac{1}{2}|\vec{\kappa}|^2\vec{\kappa}, \tag{1.2}$$

where $\nabla_{s_f}\phi := \partial_{s_f}\phi - \langle \partial_{s_f}\phi, \partial_{s_f} f\rangle \partial_{s_f} f$, see for instance [11, Lemma A.1].

Since the energy $\mathcal{E}$ might be decreased by letting the curve grow towards infinity (just think of a (portion of a) circle whose radius is expanding), it is typical to penalize the growth of the length of the curve by considering the functional

$$\mathcal{E}_\lambda(f) := \mathcal{E}(f) + \lambda\mathcal{L}(f) \tag{1.3}$$

for a given positive constant λ. The L^2-gradient is then given by

$$\nabla_{L^2}\mathcal{E}_\lambda(f) = \nabla_{s_f}^2\vec{\kappa} + \frac{1}{2}|\vec{\kappa}|^2\vec{\kappa} - \lambda\vec{\kappa}, \tag{1.4}$$

and the associated evolution reads

$$\begin{cases} \partial_t f = -\nabla_{s_f}^2\vec{\kappa} - \frac{1}{2}|\vec{\kappa}|^2\vec{\kappa} + \lambda\vec{\kappa} & t \in (0, T), \\ f(t, 0) = f_-, \quad f(t, 1) = f_+ & t \in (0, T), \\ \partial_{s_f} f(t, 0) = T_-, \quad \partial_{s_f} f(t, 1) = T_+ & t \in (0, T), \\ f(0, \cdot) = f_0(\cdot) & \end{cases} \tag{1.5}$$

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