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# Global existence and minimal decay regularity for the Timoshenko system: The case of non-equal wave speeds

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**Abstract**

As a continued work of [18], we are concerned with the Timoshenko system in the case of non-equal wave speeds, which admits the dissipative structure of *regularity-loss*. Firstly, with the modification of a priori estimates in [18], we construct global solutions to the Timoshenko system pertaining to data in the Besov space with the regularity $s = 3/2$. Owing to the weaker dissipative mechanism, extra higher regularity than that for the global-in-time existence is usually imposed to obtain the optimal decay rates of classical solutions, so it is almost impossible to obtain the optimal decay rates in the critical space. To overcome the outstanding difficulty, we develop a new frequency-localization time-decay inequality, which captures the information related to the integrability at the high-frequency part. Furthermore, by the energy approach in terms of high-frequency and low-frequency decomposition, we show the optimal decay rate for Timoshenko system in critical Besov spaces, which improves previous works greatly.


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## 1. Introduction

In this work, we are concerned with the following Timoshenko system (see [28,29]), which is a set of two coupled wave equations of the form

$$\begin{cases} \varphi_{tt} - (\varphi_x - \psi)_x = 0, \\ \psi_{tt} - \sigma(\psi_x)_x - (\varphi_x - \psi) + \gamma \psi_t = 0. \end{cases} \tag{1.1}$$

System (1.1) describes the transverse vibrations of a beam. Here, $t \geq 0$ is the time variable, $x \in \mathbb{R}$ is the spacial variable which denotes the point on the center line of the beam, $\varphi(t, x)$ is the transversal displacement of the beam from an equilibrium state, and $\psi(t, x)$ is the rotation angle of the filament of the beam. The smooth function $\sigma(\eta)$ satisfies $\sigma'(\eta) > 0$ for any $\eta \in \mathbb{R}$, and $\gamma$ is a positive constant. We focus on the Cauchy problem of (1.1), so the initial data are supplemented as

$$(\varphi, \varphi_t, \psi, \psi_t)(x, 0) = (\varphi_0, \varphi_1, \psi_0, \psi_1)(x). \tag{1.2}$$

Based on the change of variable introduced by Ide, Haramoto, and the third author [11]:

$$v = \varphi_x - \psi, \quad u = \varphi_t, \quad z = a\psi_x, \quad y = \psi_t, \tag{1.3}$$

with $a > 0$ being the sound speed defined by $a^2 = \sigma'(0)$, it is convenient to rewrite (1.1)–(1.2) as a Cauchy problem for the first-order hyperbolic system of $U = (v, u, z, y)^\top$

$$\begin{cases} U_t + A(U)U_x + LU = 0, \\ U(x, 0) = U_0(x) \end{cases} \tag{1.4}$$

with $U_0(x) = (v_0, u_0, z_0, y_0)(x)$, where $v_0 = \varphi_{0,x} - \psi_0$, $u_0 = \varphi_1$, $z_0 = a\psi_{0,x}$, $y_0 = \psi_1$ and

$$A(U) = -\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a \\ 0 & 0 & \frac{\sigma'(z/a)}{a} & 0 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & \gamma \end{pmatrix}.$$

Note that $A(U)$ is a real symmetrizable matrix due to $\sigma'(z/a) > 0$, and the dissipative matrix $L$ is nonnegative definite but not symmetric. Such degenerate dissipation forces (1.4) to go beyond the class of generally dissipative hyperbolic systems, so the recent global-in-time existence (see [31]) for hyperbolic systems with symmetric dissipation cannot be applied directly, which is the main motivation on studying the Timoshenko system (1.1).

Let us review several known results on (1.1). In a bounded domain, it is well-known that (1.1) is exponentially stable if the damping term $\varphi_t$ is also present on the left-hand side of the first equation of (1.3) (see, e.g., [21]). Soufyane [27] showed that (1.1) could not be exponentially stable by considering only the damping term of the form $\psi_t$, unless for the case of $a = 1$ (equal wave speeds). A similar result was obtained by Rivera and Racke [23] with an alternative proof.