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Dense heteroclinic tangencies near a Bykov cycle

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Abstract

This article presents a mechanism for the coexistence of hyperbolic and non-hyperbolic dynamics arising in a neighbourhood of a Bykov cycle where trajectories turn in opposite directions near the two nodes — we say that the nodes have different chirality. We show that in the set of vector fields defined on a three-dimensional manifold, there is a class where tangencies of the invariant manifolds of two hyperbolic saddle-foci occur densely. The class is defined by the presence of the Bykov cycle, and by a condition on the parameters that determine the linear part of the vector field at the equilibria. This has important consequences: the global dynamics is persistently dominated by heteroclinic tangencies and by Newhouse phenomena, coexisting with hyperbolic dynamics arising from transversality. The coexistence gives rise to linked suspensions of Cantor sets, with hyperbolic and non-hyperbolic dynamics, in contrast with the case where the nodes have the same chirality.

We illustrate our theory with an explicit example where tangencies arise in the unfolding of a symmetric vector field on the three-dimensional sphere.

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Keywords: Heteroclinic cycle; Heteroclinic tangencies; Non-hyperbolicity; Chirality; Quasi-stochastic attractor

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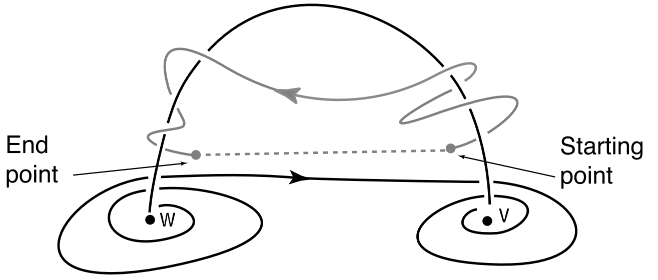

Fig. 1. Bykov cycle with saddle-foci of different chirality. The starting point of a nearby trajectory is joined to its end point forming a loop. Arbitrarily close to the cycle there are trajectories whose loop is not linked to the cycle. This happens because near each saddle-focus trajectories turn around the one-dimensional connection in opposite directions.

1. Introduction

Consider a differential equation in a three-dimensional manifold having a heteroclinic cycle that consists of two saddle-foci of different Morse indices whose one-dimensional invariant manifolds coincide and whose two-dimensional invariant manifolds intersect transversely. There are two different possibilities for the geometry of the flow around the cycle, depending on the direction trajectories turn around the heteroclinic connection of one-dimensional invariant manifolds. The two cases give rise to different dynamics, but the distinction is usually not made explicitly in the literature. This article is concerned with the case when, near the two saddle-foci, trajectories wind in opposite directions around the heteroclinic connection of one-dimensional invariant manifolds — the two nodes have *different chirality* as in Fig. 1.

The dynamics around this type of cycle was first studied by V.V. Bykov [7,8], with the implicit assumption of different chirality. He has obtained an open class containing a dense set of vector fields exhibiting tangencies of the two-dimensional invariant manifolds — see also [20, Th. 5.33]. Bykov also described bifurcations occurring when the structurally unstable one-dimensional connection is broken.

In the present article we highlight that the orientation of the flow around the structurally unstable connection has profound effects on the dynamics near the cycle. We refine and clarify the key ideas of the analysis of the unperturbed system of [7,8] and we explore properties of the maximal invariant set that emerges near the Bykov cycle. After recovering Bykov's one-pulse heteroclinic tangencies, we show the existence of multi-pulse connections, occurring along trajectories that follow the original cycle an arbitrary number of times. We also show that the non-hyperbolic set containing heteroclinic tangencies coexists with the suspension of uniformly hyperbolic horseshoes. Although each individual tangency may be eliminated by a small perturbation, another tangency is created nearby, while the hyperbolic set persists. By construction, the single-round periodic solutions and the single-round heteroclinic trajectories found in [8, Th. 3.1 and 3.2] lie inside the suspended horseshoes found here. An explicit vector field where the nodes have different chirality is also constructed here.

Tangencies of invariant manifolds are associated with Newhouse phenomena: bifurcations leading to the birth of infinitely many asymptotically stable periodic solutions [32,33]. Such tangencies have been recognised as a mechanism for instability and lack of hyperbolicity in surface diffeomorphisms. One of the first results in this direction was established by Gavrilov and Shilnikov [12] for two-dimensional maps. To understand this phenomenon it is important to study the variety of dynamical behaviour associated with the creation and destruction of tan-