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Classical solutions for the ellipsoidal BGK model with fixed collision frequency

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Abstract

We establish the existence of global in time smooth solutions for the ellipsoidal BGK model, which is a variant of the BGK model for the Boltzmann equation designed to yield the correct Prandtl number in the hydrodynamic approximation at the Navier–Stokes level. For this, we carefully design a function space which captures the growth of the solution in a weighted Sobolev norm, and show that the ellipsoidal relaxation operator is Lipschitz continuous in the induced metric. This approach is restricted to the case when the collision frequency does not depend on the macroscopic field, but no smallness on the initial data is required.

Keywords: Ellipsoidal BGK model; Boltzmann equation; Prandtl number; Non-isotropic Gaussian; Classical solutions

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1. Introduction

The BGK model is widely used in science and engineering in place of the Boltzmann equation since it reproduces various qualitative properties of the Boltzmann equation at a much lower computational cost. But it's been known for a long time that the BGK approximation of the Navier Stokes equation does not give the correct Prandtl number, which is defined as the ratio between the viscosity and the thermal conductivity. In an effort to cook up a variant of the BGK model that gives the correct Prandtl number, Holway introduced the ellipsoidal BGK model (ES-BGK model) [19]:

$$\begin{aligned} &\partial_t f + v\cdot\nabla f = A_\nu(\mathcal{M}_\nu(f) - f), \quad (x,v,t)\in\mathbb{T}^3\times\mathbb{R}^3\times\mathbb{R}_+,\\ &f(x,v,0) = f_0(x,v). \end{aligned} \tag{1.1}$$

The particle distribution function $f(x,v,t)$ denotes the number density of particles on the phase space point $(x,v)\in\mathbb{T}^3\times\mathbb{R}^3$ at time $t\in\mathbb{R}^+$. The non-isotropic Gaussian $\mathcal{M}_\nu(f)$ takes the following form

$$\mathcal{M}_\nu(f) = \frac{\rho}{\sqrt{\det(2\pi\mathcal{T}_\nu)}}\exp\left(-\frac{1}{2}(v-U)^\top\mathcal{T}_\nu^{-1}(v-U)\right),$$

where ρ, U, T, Θ are the macroscopic density, bulk velocity, temperature, stress tensor respectively:

$$\begin{aligned} \rho(x,t) &= \int_{\mathbb{R}^3} f(x,v,t)dv,\\ \rho(x,t)U(x,t) &= \int_{\mathbb{R}^3} f(x,v,t)v\,dv,\\ 3\rho(x,t)T(x,t) &= \int_{\mathbb{R}^3} f(x,v,t)|v-U(x,t)|^2dv,\\ \rho(x,t)\Theta(x,t) &= \int_{\mathbb{R}^3} f(x,v,t)(v-U)\otimes(v-U)dv\\ &= \int_{\mathbb{R}^3} F(x,v,t)(v\otimes v)dv - \rho U\otimes U, \end{aligned}$$