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Persistence of mass in a chemotaxis system with logistic source

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Abstract

This paper studies the dynamical properties of the chemotaxis system

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + ru - \mu u^2, & x\in\Omega,\ t>0,\\ v_t = \Delta v - v + u, & x\in\Omega,\ t>0, \end{cases} \qquad (\star)$$

under homogeneous Neumann boundary conditions in bounded convex domains $\Omega\subset\mathbb{R}^n$, $n\geq 1$, with positive constants χ, r and μ.

Numerical simulations but also some rigorous evidence have shown that depending on the relative size of r, μ and $|\Omega|$, in comparison to the well-understood case when $\chi=0$, this problem may exhibit quite a complex solution behavior, including unexpected effects such as asymptotic decay of the quantity u within large subdomains of Ω.

The present work indicates that any such extinction phenomenon, if occurring at all, necessarily must be of spatially local nature, whereas the population as a whole always persists. More precisely, it is shown that for any nonnegative global classical solution (u,v) of $(\star)$ with $u\not\equiv 0$ one can find $m_\star>0$ such that

$$\int_\Omega u(x,t)dx \geq m_\star \qquad \text{for all } t>0.$$

The proof is based on an, in this context, apparently novel analysis of the functional $\int_\Omega \ln u$, deriving a lower bound for this quantity along a suitable sequence of times by appropriately exploiting a differential inequality for a suitable linear combination of $\int_\Omega \ln u$, $\int_\Omega u$ and $\int_\Omega v^2$.

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1. Introduction

Chemotaxis, a biological process in which cells migrate towards higher concentrations of a chemical signal, has received great interest in biological and mathematical communities. A celebrated model for such processes, as introduced by Keller and Segel in 1970 [8], consists of two parabolic equations reflecting chemotactic movement through a nonlinear cross-diffusive term as their most characteristic ingredient. Since then, considerable efforts have been undertaken to comprehend possible effects of this interaction in various frameworks, with the detection of finite-time blow-up in the classical Keller–Segel system constituting the apparently most striking evidence for the strongly destabilizing action of chemotactic cross-diffusion [7,11,23]. Accordingly, large bodies of the literature focus on identifying circumstances under which either global bounded solutions can be constructed, or explosions occur.

In contrast to the rich knowledge on this basic issue, understanding the qualitative properties even of bounded solutions to chemotaxis problems seems much less developed. Genuinely parabolic features such as convergence to single equilibria could up to now only be verified in some particular cases when either an appropriate entropy-dissipation structure inhibits oscillatory behavior [3,17], or when some negligibility of cross-diffusion as compared to diffusion is enforced by certain smallness assumptions on the initial data or on parameters measuring the strength of chemotaxis [21,2,24,19].

In the present work we address a dynamical property of the Keller–Segel system with logistic source given by

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + ru - \mu u^2, & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega,\ t > 0, \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

in bounded convex domains $\Omega \subset \mathbb{R}^n$, $n \geq 1$, with positive constants χ, r and μ, where as usual u denotes the cell density and v represents the concentration of a signal produced by cells themselves. Here the cell kinetic term on the one hand exerts a certain growth-inhibiting influence; indeed, in the case $n \leq 2$ even arbitrarily small $\mu > 0$ are sufficient to rule out any explosion by guaranteeing global existence of bounded classical solutions for all reasonably smooth initial data [1,14], whereas in the case $n \geq 3$ the same conclusion holds provided that $\mu > 0$ is suitably large [20]. On the other hand, this additional logistic term apparently destroys the well-known energy structure of the corresponding free Keller–Segel system obtained in the limit case $r = \mu = 0$ [12].

Apparently, however, the latter does not only reduce the accessibility of (1.1) to convenient mathematical tools, but beyond this it reflects a substantial change in dynamical complexity. Indeed, numerical evidence shows that even in the spatially one-dimensional setting solutions