Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations 259 (2015) 6459–6493

Journal of Differential Equations

www.elsevier.com/locate/jde

Stabilization of a fluid–rigid body system

Takéo Takahashi [a,b], Marius Tucsnak [c,*], George Weiss [d]

[a] Inria, Villers-lès-Nancy F-54600, France

[b] University of Lorraine, Vandœuvre-lès-Nancy 54506, France

[c] Institut Elie Cartan, University of Lorraine, Vandoeuvre les Nancy 54506, France

[d] Department of EE-Systems, Tel Aviv University, Ramat Aviv 69978, Israel

Received 1 October 2014; revised 2 July 2015

Available online 7 August 2015

Abstract

We consider the mathematical model of a rigid ball moving in a viscous incompressible fluid occupying a bounded domain Ω, with an external force acting on the ball. We investigate in particular the case when the external force is what would be produced by a spring and a damper connecting the center of the ball h to a fixed point $h_1 \in \Omega$. If the initial fluid velocity is sufficiently small, and the initial h is sufficiently close to h_1, then we prove the existence and uniqueness of global (in time) solutions for the model. Moreover, in this case, we show that h converges to h_1, and all the velocities (of the fluid and of the ball) converge to zero. Based on this result, we derive a control law that will bring the ball asymptotically to the desired position h_1 even if the initial value of h is far from h_1, and the path leading to h_1 is winding and complicated. Now, the idea is to use the force as described above, with one end of the spring and damper at h, while other end is jumping between a finite number of points in Ω, that depend on h (a switching feedback law).

MSC: 35Q35; 35D05; 35Q30; 35Q72; 76D03

Keywords: Fluid–structure interactions; Navier–Stokes equations; PD controller; Global solutions; Asymptotic stability; Switching feedback

* Corresponding author.

E-mail addresses: takeo.takahashi@inria.fr (T. Takahashi), Marius.Tucsnak@univ-lorraine.fr (M. Tucsnak), gweiss@eng.tau.ac.il (G. Weiss).

http://dx.doi.org/10.1016/j.jde.2015.07.024

1. Introduction and main results

We consider a coupled system described by nonlinear partial and ordinary differential equations modelling the motion of a rigid body inside a viscous incompressible fluid in a bounded domain Ω. The fluid flow is described by the classical *Navier–Stokes equations* (see (1.1)–(1.2) below), whereas the motion of the ball-shaped rigid body is governed by the *Newton laws* (see (1.6)–(1.7) below), including an external control force denoted by u acting on the ball.

The domain occupied by the fluid and the rigid ball is $\Omega \subset \mathbb{R}^3$, a connected open bounded set with C^2 boundary. The rigid ball has radius 1 and its center is located at the (variable) point h which is at a distance > 1 from the boundary $\partial\Omega$. We denote by $\mathcal{B}(h)$ the closed set occupied by the ball. The fluid is homogeneous with *density* $\rho > 0$ and *viscosity* $\nu > 0$ and it occupies the domain

$$\mathcal{F}(h) = \Omega \setminus \mathcal{B}(h).$$

The full system of equations modelling the system, for $t \geqslant 0$, is

$$\rho\dot{v} - \nu\Delta v + \rho(v \cdot \nabla)v + \nabla p = 0, \quad x \in \mathcal{F}(h(t)), \tag{1.1}$$

$$\operatorname{div} v = 0, \quad x \in \mathcal{F}(h(t)), \tag{1.2}$$

$$v = 0, \quad x \in \partial\Omega, \tag{1.3}$$

$$\dot{h} = g, \tag{1.4}$$

$$v = g(t) + \omega(t) \times (x - h(t)), \quad x \in \partial\mathcal{B}(h(t)), \tag{1.5}$$

$$m\dot{g} = -\int_{\partial\mathcal{B}(h)} \sigma(v, p)n \,\mathrm{d}\Gamma + u, \tag{1.6}$$

$$J\dot{\omega} = -\int_{\partial\mathcal{B}(h)} (x - h) \times \sigma(v, p)n \,\mathrm{d}\Gamma, \tag{1.7}$$

$$h(0) = h_0, \quad \dot{h}(0) = g_0, \quad \omega(0) = \omega_0, \tag{1.8}$$

$$v(x, 0) = v_0(x), \qquad x \in \mathcal{F}(h_0). \tag{1.9}$$

In the above system the *state variables* are $v(x,t)$ (the Eulerian velocity field of the fluid), $h(t)$ (the position of the center of the rigid ball), its time derivative $g(t)$, and $\omega(t)$ (the angular velocity of the ball). The function $p(x,t)$ is the pressure of the fluid, which is not a state variable, because at any time instant it can be computed from v at the same instant, up to an additive constant. We have denoted by $n(x,t)$ the unit normal to $\partial\mathcal{B}(h(t))$ at the point $x \in \partial\mathcal{B}(h(t))$, directed to the interior of the ball, and by m and J the mass and the moment of inertia of the rigid ball. (If we would take $\nu = 0$, then (1.1)–(1.2) would be called *Euler's equations*, but then the other equations and the nature of the system would change.) We have denoted by $\sigma(v, p)$ the tensor defined by

$$\sigma_{ij}(v, p) = -p\delta_{ij} + \nu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \qquad (i,\ j \in \{1, 2, 3\}). \tag{1.10}$$