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Global existence of classical solutions for isentropic compressible Navier–Stokes equations with small initial density ☆

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Abstract

In this paper we establish the global existence of classical solutions to the Cauchy problem for the 3-D isentropic compressible Navier–Stokes equations with smooth initial data that are small density but possibly large energy, which could be either vacuum or non-vacuum.

MSC: 76N10; 35M10

Keywords: Compressible N-S equations; Small initial density; Global classical solutions

1. Introduction

The motion of a viscous isentropic compressible fluid occupying a domain $\Omega \subset \mathbb{R}^3$ is governed by the compressible Navier–Stokes equations

$$\begin{cases} \rho_t + \mathrm{div}(\rho u) = 0, \\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) - \mu \triangle u - (\mu + \lambda)\nabla(\mathrm{div} u) + \nabla P(\rho) = 0, \end{cases} \tag{1.1}$$

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where $\rho \geq 0$, $u = (u^1, u^2, u^3)$ and $P = a\rho^\gamma$ $(a > 0, \gamma > 1)$ are the fluid density, velocity and pressure, respectively. The constant viscosity coefficients μ and λ satisfy the physical restrictions

$$\mu > 0,\ \mu + \frac{3}{2}\lambda \geq 0. \tag{1.2}$$

Let $\Omega = \mathbb{R}^3$. We look for the solutions, $(\rho(x,t), u(x,t))$, to the Cauchy problem for (1.1) with the far field behavior:

$$u(x,t) \to 0,\ \rho(x,t) \to 0,\ \text{as } |x| \to \infty, \tag{1.3}$$

and initial data,

$$(\rho, u)|_{t=0} = (\rho_0, u_0),\ x \in \mathbb{R}^3. \tag{1.4}$$

Much efforts have been devoted to study the global existence and behavior of solutions to (1.1). The one dimensional problem has been studied extensively by many people (see [1–4] and the references therein). For the multi-dimensional case, the local existence and uniqueness of classical solutions are known in [3,5–9] in the absence of vacuum and in [1,10–12] for the case that the initial density need not be positive and may vanish in an open set. The global classical solutions were first obtained by Matsumura and Nishida [13] for initial data close to a nonvacuum equilibrium in some Sobolev space H^s. Later, Hoff [10,11] studied the problem for discontinuous initial data. For the existence of solutions for arbitrary data (the far field is vacuum, that is $\tilde{\rho} = 0$), the major breakthough is due to Lions [14] (see also Feireisl [15]), where he proved the global existence of weak solutions, defined as solutions with finite-energy, when the adiabatic exponent γ is suitably large (i.e. $\gamma > 3/2$). The main restriction on initial data is that the initial energy is finite, so that, the density vanishes at far fields, or even has compact support. However, the uniqueness and regularity of such weak solutions are still open. Recently, Huang, Li and Xin [16] establish the global well-posedness of classical solutions with large oscillations and vacuum to the Cauchy problem (1.1)–(1.4) under the assumption that the initial energy is suitably small. The result obtained in [16] is an important advance in the study of compressible Navier–Stokes equations. Lately using the idea in [16], Deng, Zhang and Zhao [17] establish the global existence and uniqueness of classical solutions to the Cauchy problem of (1.1)–(1.4) under the assumption that the viscosity coefficient μ is large enough.

In this paper we are interested in studying the global existence of classical solutions to Cauchy problem (1.1)–(1.4) with large initial energy, which could be either vacuum or non-vacuum. Basing on the ideals in [16], we establish the global existence of classical solutions with general initial energy under the assumption that the upper bound of the initial density is suitably small.

Before stating the main results, we explain the notations and conventions used throughout this paper. For simplicity we set

$$\int f dx = \int_{\mathbb{R}^3} f dx.$$