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J. Differential Equations 259 (2015) 2388–2407

Journal of Differential Equations

www.elsevier.com/locate/jde

Diffusion limit of 3D primitive equations of the large-scale ocean under fast oscillating random force

Boling Guo [a], Daiwen Huang [a,*], Wei Wang [b]

[a] Institute of Applied Physics and Computational Mathematics, Beijing 100088, China
[b] Department of Mathematics, Nanjing University, Nanjing 210093, China

Received 31 August 2013; revised 23 December 2014

Available online 22 April 2015

Abstract

The three-dimensional (3D) viscous primitive equations describing the large-scale oceanic motions under fast oscillating random perturbation are studied. Under some assumptions on the random force, the solution to the initial boundary value problem (IBVP) of the 3D random primitive equations converges in distribution to that of IBVP of the limiting equations, which are the 3D stochastic primitive equations describing the large-scale oceanic motions under a white in time noise forcing. This also implies the convergence of the stationary solution of the 3D random primitive equations.

Keywords: Random primitive equations; Stationary solution; Martingale; Statistical solution

1. Introduction

The important 3D viscous primitive equations of the large-scale ocean in a Cartesian coordinate system, are written as the following system on a cylindrical domain

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v + \Phi(v)\frac{\partial v}{\partial z} + fk \times v + \nabla p_b - \int_{-1}^{z} \nabla T dz'$$

* Corresponding author.
E-mail addresses: hdw55@tom.com, huang_daiwen@iapcm.ac.cn (D. Huang).

http://dx.doi.org/10.1016/j.jde.2015.03.041

$$-\Delta v-\frac{\partial^2 v}{\partial z^2}=\Psi_1, \tag{1.1}$$

$$\frac{\partial T}{\partial t}+(v\cdot\nabla)T+\Phi(v)\frac{\partial T}{\partial z}-\triangle T-\frac{\partial^2 T}{\partial z^2}=\Psi_2, \tag{1.2}$$

$$\int_{-1}^{0}\nabla\cdot v\,dz=0, \tag{1.3}$$

with boundary value conditions

$$\frac{\partial v}{\partial z}=0,\quad \frac{\partial T}{\partial z}=-\alpha_u T \qquad \text{on} \quad M\times\{0\}=\Gamma_u, \tag{1.4}$$

$$\frac{\partial v}{\partial z}=0,\quad \frac{\partial T}{\partial z}=0 \qquad \text{on} \quad M\times\{-1\}=\Gamma_b, \tag{1.5}$$

$$v\cdot\vec{n}=0,\quad \frac{\partial v}{\partial\vec{n}}\times\vec{n}=0,\quad \frac{\partial T}{\partial\vec{n}}=0 \quad \text{on } \partial M\times[-1,0]=\Gamma_l, \tag{1.6}$$

and the initial value conditions

$$u|_{t=t_0}=(v|_{t=t_0},T|_{t=t_0})=u_{t_0}=(v_{t_0},T_{t_0}), \tag{1.7}$$

where the unknown functions are v, p_b, T, $v=(v^{(1)},v^{(2)})$ the horizontal velocity, p_b the pressure, T temperature, $\Phi(v)(t,x,y,z)=-\int_{-1}^{z}\nabla\cdot v(t,x,y,z')\,dz'$ vertical velocity, $f=f_0(\beta+y)$ the Coriolis parameter, k vertical unit vector, Ψ_1 a given forcing field, Ψ_2 a given heat source, $\nabla=(\partial_x,\ \partial_y)$, $\triangle=\partial_x^2+\partial_y^2$, α_u a positive constant, $\vec{n}$ the norm vector to Γ_l and M a smooth bounded domain in $\mathbb{R}^2$. For more details for (1.1)–(1.7), see [2,21] and the references therein.

In the past two decades, there were several research works about the well-posedness of the above 3D deterministic primitive equations of the large-scale ocean. In [17], Lions, Temam and Wang obtained the global existence of weak solutions for the primitive equations. In [14], Guillén-González etc. obtained the global existence of strong solutions to the primitive equations with small initial data. Moreover, they proved the local existence of strong solutions to the equations. In [2], Cao and Titi developed a beautiful approach to proving that L^6-norm of the horizontal velocity is uniformly in t bounded, and obtained the global well-posedness for the 3D viscous primitive equations.

In study of the primitive equations of the large-scale ocean or atmosphere, taking the stochastic external factors into account is reasonable and necessary. There are many works about mathematical study of some stochastic climate models, see, e.g., [6–8,19,20]. Ref. [8] is one of the first works on a 3D stochastic quasi-geostrophic model. Guo and Huang in [13] considered the global well-posedness and long-time dynamics for the 3D stochastic primitive equations of the large-scale ocean under a white in time noise forcing.

In realistic model, random fluctuation always exits. We consider the following 3D primitive equations with fast oscillating random force