

On the radius of spatial analyticity for the 1d Dirac–Klein–Gordon equations

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Abstract

We study the well-posedness of the Dirac–Klein–Gordon system in one space dimension with initial data that have an analytic extension to a strip around the real axis. It is proved that the radius of analyticity $\sigma(t)$ of the solutions at time t cannot decay faster than $1/t^4$ as $|t| \rightarrow \infty$.

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1. Introduction

Consider the Dirac–Klein–Gordon equations (DKG) on $\mathbb{R}^{1+1}$,

$$\begin{cases} (-i\gamma^0\partial_t - i\gamma^1\partial_x + M)\psi = \phi\psi, \\ (\partial_t^2 - \partial_x^2 + m^2)\phi = \psi^*\gamma^0\psi, \end{cases} \quad (t, x \in \mathbb{R}) \quad (1)$$

with initial condition

$$\psi(0, x) = \psi_0(x), \quad \phi(0, x) = \phi_0(x), \quad \partial_t\phi(0, x) = \phi_1(0, x). \quad (2)$$

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Here the unknowns are $\phi: \mathbb{R}^{1+1} \rightarrow \mathbb{R}$ and $\psi: \mathbb{R}^{1+1} \rightarrow \mathbb{C}^2$, the latter regarded as a column vector with conjugate transpose ψ^* . The masses $M, m \geq 0$ are given constants. The 2×2 Dirac matrices γ^0, γ^1 should satisfy $\gamma^0 \gamma^1 + \gamma^1 \gamma^0 = 0$, $(\gamma^0)^2 = I$, $(\gamma^1)^2 = -I$, $(\gamma^0)^* = \gamma^0$ and $(\gamma^1)^* = -\gamma^1$; we will work with the representation

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The well-posedness of this Cauchy problem with data in the family of Sobolev spaces $H^s = (1 - \partial_x^2)^{-s/2} L^2(\mathbb{R})$, $s \in \mathbb{R}$, has been intensively studied; see [8,4,2,5,18,15,21,20,19,23,16,22,6]. Local well-posedness holds for data

$$(\psi_0, \phi_0, \phi_1) \in H^s(\mathbb{R}; \mathbb{C}^2) \times H^r(\mathbb{R}; \mathbb{R}) \times H^{r-1}(\mathbb{R}; \mathbb{R}) \quad (3)$$

with $s > -1/2$ and $|s| \leq r \leq s+1$; see [16], where it is also proved that this is the optimal result, in the sense that for other (r, s) either one has ill-posedness or the solution map (if it exists) is not regular. Moreover, when $s \geq 0$ there is conservation of charge,

$$\|\psi(t)\|_{L^2} = \|\psi_0\|_{L^2},$$

implying that the solutions extend globally when $0 \leq s \leq r \leq s+1$, and by propagation of higher regularity the global solution is C^∞ if the data are C^∞ .

While the well-posedness in Sobolev spaces is well-understood, much less is known concerning spatial analyticity of the solutions to the above Cauchy problem, and this is what motivates the present paper.

On the one hand, local propagation of analyticity for nonlinear hyperbolic systems has been studied by Alinhac and Métivier [1] and Jannelli [11], and in particular this general theory implies that if the data (3) (with s, r sufficiently large) are analytic on the real line, then the same is true of the solution $(\psi, \phi, \partial_t \phi)(t)$ to (1) for all times t . The local theory does not give any information about the radius of analyticity, however.

On the other hand, one can consider the situation where a uniform radius of analyticity on the real line is assumed for the initial data, so there is a holomorphic extension to a strip $\{x + iy: |y| < \sigma_0\}$ for some $\sigma_0 > 0$. One may then ask whether this property persists for all later times t , but with a possibly smaller and shrinking radius of analyticity $\sigma(t) > 0$. This type of question was introduced in an abstract setting of nonlinear evolutionary PDE by Kato and Masuda [12], who showed in particular that for the Korteweg–de Vries equation (KdV) the radius of analyticity $\sigma(t)$ can decay to zero at most at a super-exponential rate. A similar rate of decay for semilinear symmetric hyperbolic systems has been proved recently by Cappelletto, D’Ancona and Nicola [7]. An algebraic rate of decay for KdV was shown by Bona, Grujić and Kalisch [3]. Panizzi [17] has obtained an algebraic rate for nonlinear Klein–Gordon equations. In this paper our aim is to obtain an algebraic rate for the DKG system.

We use the following spaces of Gevrey type. For $\sigma \geq 0$ and $s \in \mathbb{R}$, let $G^{\sigma,s}$ be the Banach space with norm

$$\|f\|_{G^{\sigma,s}} = \|e^{\sigma|\xi|} \langle \xi \rangle^s \widehat{f}(\xi)\|_{L^2_\xi},$$

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