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Regularity analysis for an abstract system of coupled hyperbolic and parabolic equations

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Abstract

In this paper, we provide a complete regularity analysis for the following abstract system of coupled hyperbolic and parabolic equations

$$\begin{cases} u_{tt} = -Au + \gamma A^{\alpha} w, \\ w_t = -\gamma A^{\alpha} u_t - kA^{\beta} w, \\ u(0) = u_0, \quad u_t(0) = v_0, \quad w(0) = w_0, \end{cases}$$

where A is a self-adjoint, positive definite operator on a complex Hilbert space H, and $(\alpha, \beta) \in [0,1] \times [0,1]$. We are able to decompose the unit square of the parameter (α, β) into three parts where the semigroup associated with the system is analytic, of specific order *Gevrey* classes, and non-smoothing, respectively. Moreover, we will show that the orders of *Gevrey* class is sharp, under proper conditions.

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Keywords: Hyperbolic–parabolic equations; Analytic semigroup; *Gevrey* class semigroup

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1. Introduction

Let H be a complex Hilbert space with the inner product $\langle\cdot,\cdot\rangle$ and the induced norm $\|\cdot\|$. We consider the following abstract system of coupled hyperbolic and parabolic equations:

$$\begin{cases} u_{tt} = -Au + \gamma A^\alpha w, \\ w_t = -\gamma A^\alpha u_t - kA^\beta w, \\ u(0) = u_0, \quad u_t(0) = v_0, \quad w(0) = w_0, \end{cases} \tag{1.1}$$

where A is a self-adjoint, positive definite (unbounded) operator on a complex Hilbert space H; $\gamma \neq 0$, $k > 0$, and $\alpha, \beta \in [0,1]$ are fixed real numbers. Our main interest is the regularity of the solution to this system in terms of the parameters α, β.

We define

$$\mathcal{H} = \mathcal{D}(A^{\frac{1}{2}}) \times H \times H.$$

Any element in $\mathcal{H}$ is denoted by $U = (u, v, w)^T$. Introduce

$$\langle U_1, U_2\rangle_{\mathcal{H}} = \langle A^{\frac{1}{2}}u_1, A^{\frac{1}{2}}u_2\rangle + \langle v_1, v_2\rangle + \langle w_1, w_2\rangle, \qquad \forall\, U_i = \begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \in \mathcal{H},\ i = 1, 2.$$

Then $\langle\cdot,\cdot\rangle_{\mathcal{H}}$ is an inner product under which $\mathcal{H}$ is a Hilbert space. By denoting $v = u_t$ and $U_0 = (u_0, v_0, w_0)^T$, system (1.1) can be written as an abstract linear evolution equation on the space $\mathcal{H}$,

$$\begin{cases} \dfrac{dU(t)}{dt} = \mathcal{A}_{\alpha,\beta} U(t), \qquad t \ge 0, \\ U(0) = U_0, \end{cases} \tag{1.2}$$

where the operator $\mathcal{A}_{\alpha,\beta} : \mathcal{D}(\mathcal{A}_{\alpha,\beta}) \subseteq \mathcal{H} \to \mathcal{H}$ is defined by

$$\mathcal{A}_{\alpha,\beta} = \begin{pmatrix} 0 & I & 0 \\ -A & 0 & \gamma A^\alpha \\ 0 & -\gamma A^\alpha & -kA^\beta \end{pmatrix}, \tag{1.3}$$

with the domain

$$\mathcal{D}(\mathcal{A}_{\alpha,\beta}) = \mathcal{D}(A) \times \mathcal{D}(A^{\alpha \vee \frac{1}{2}}) \times \mathcal{D}(A^{\alpha \vee \beta}), \tag{1.4}$$

where $a \vee b = \max\{a, b\}$ for any $a, b \in \mathbb{R}$. It is known that $\mathcal{A}_{\alpha,\beta}$ (which is identified with its closure) generates a C_0-semigroup $e^{\mathcal{A}_{\alpha,\beta}t}$ of contractions on $\mathcal{H}$ [1]. Then the solution to the evolution equation (1.2) admits the following representation:

$$U(t) = e^{\mathcal{A}_{\alpha,\beta}t} U_0, \qquad t \ge 0,$$

which leads to the well-posedness of (1.2). With this in hand, regularity and stability are the most interesting properties for the solutions to evolution equations that attract people's attention.