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Long-time behavior of a class of thermoelastic plates with nonlinear strain

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Abstract

In recent years a class of vibrating plates with nonlinear strain of p-Laplacian type was studied by several authors. The present paper contains a first thermoelastic model of that class of problems including both Fourier and non-Fourier heat laws. Our main result establishes the existence of global and exponential attractors for the strongly damped problem through a stabilizability inequality. In addition, for the weakly damped problem, we establish the exponential stability of its Galerkin semiflows.

Keywords: Thermoelastic plates; Thermal memory; Global attractors; p-Laplacian; Kirchhoff models

1. Introduction

In recent years several authors have studied the asymptotic behavior of problems of the form

$$u_{tt} + \Delta^2 u = \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \sigma_i \left(u_{x_i} \right) + \{\text{damping and forcing}\} \text{ in } \Omega \times \mathbb{R}^+, \tag{1.1}$$

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where Ω is a bounded domain of $\mathbb{R}^N$ and σ_i are polynomial like. They mainly considered the operator in the right hand side of (1.1) as a version of p-Laplacian. Indeed, when

$$\sigma_i\left(u_{x_i}\right) = |\nabla u|^{p-2} u_{x_i} \quad \text{or} \quad \sigma_i\left(u_{x_i}\right) = |u_{x_i}|^{p-2} u_{x_i},$$

we have precisely the following usual forms for the p-Laplacian operator,

$$\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u) \quad \text{or} \quad \Delta_p u = \sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left(\left|\frac{\partial u}{\partial x_i}\right|^{p-2} \frac{\partial u}{\partial x_i}\right).$$

Evolution problems modeled on equation (1.1) are used in many applications. In the one-dimensional case this problem is related to a model of elastoplastic microstructure and was studied by An and Peirce [1] and Yang [34,35]. In the two-dimensional case the problem is related to a class of Kirchhoff–Boussinesq models studied by Chueshov and Lasiecka [5,6,9]. In general, problem (1.1) models vibrations of elastic plates with nonlinear shear strain, as considered by Esquivel-Avila [12], Liu and Xu [26,27], Wang and Wang [32], Yang [36–38] and Yang and Jin [39]. In addition, a viscoelastic version of the problem with a memory term was proposed by Andrade, Jorge Silva and Ma [2,22,23].

Now, although the p-Laplacian appears as a lower order perturbation of the bi-harmonic operator it is not a locally Lipschitz perturbation. To this type of problem several technical difficulties arrive. For instance, from above cited works, when $N \geq 3$ a strong damping $-\Delta u_t$ is always required for the uniqueness of global solutions. If the strong damping is replaced by a weak frictional damping u_t the uniqueness is not known for $N \geq 3$. Furthermore, for $N = 2$, the uniqueness of the weakly damped problem is known with $p = 4$, as a particular case of the Kirchhoff–Boussinesq model considered by Chueshov and Lasiecka [6]. See also Ma and Pelicer [28] for the weakly damped problem in dimension $N = 1$.

To our best knowledge, problem (1.1) was not considered early in a thermoelasticity point of view. Since thermal effect is a major feature in the theory of elastic plates, our objective in the present work is to contribute with the analysis of long-time dynamics of a thermoelastic version of problem (1.1). We restrict ourselves to the two-dimensional case which is more suitable for modeling plates. Higher dimensions can be considered by adapting accordingly the growth conditions of the nonlinear terms. On the other hand we consider in a unified way Fourier and non-Fourier heat laws. Then our system reads as follows:

$$u_{tt} + \Delta^2 u - \Delta_p u - \Delta u_t + f(u) + \nu \Delta\theta = h \quad \text{in } \Omega \times \mathbb{R}^+, \tag{1.2}$$

$$\theta_t - \omega\Delta\theta - (1-\omega)\int_0^\infty k(s)\Delta\theta(t-s)ds - \nu\Delta u_t = 0 \quad \text{in } \Omega \times \mathbb{R}^+, \tag{1.3}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain with regular boundary $\partial\Omega$, $\nu > 0$ is a coupling parameter, and k is a decreasing memory kernel with $k(\infty) = 0$. Here $u = u(x,t)$ and $\theta = \theta(x,t)$ denote, respectively, the transversal displacement of the plate and the relative temperature. To the system we add boundary conditions

$$u = \Delta u = 0 \quad \text{on } \partial\Omega \times \mathbb{R}^+, \quad \theta = 0 \quad \text{on } \partial\Omega \times \mathbb{R},$$