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Asymptotic boundary estimates to infinity Laplace equations with Γ-varying nonlinearity ☆

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ABSTRACT

In a previous paper of the authors (Wang et al. (2014) [40]), the asymptotic estimates of boundary blow-up solutions were established to the infinity Laplace equation $\Delta_\infty u = b(x)f(u)$ in $\Omega \subset \mathbb{R}^N$, with the nonlinearity $0 \le f \in C[0,\infty)$ regularly varying at ∞, and the weighted function $b \in C(\bar{\Omega})$ positive in Ω and vanishing on the boundary. The present paper gives a further investigation on the asymptotic behavior of boundary blow-up solutions to the same equation by assuming f to be Γ-varying. Note that a Γ-varying function grows faster than any regularly varying function. We first quantitatively determine the boundary blow-up estimates with the first expansion, relying on the decay rate of b near the boundary and the growth rate of f at infinity, and further characterize these results via examples possessing various decay rates for b and growth rates for f. In particular, we pay attention to the second-order estimates of boundary blow-up solutions. It was observed in our previous work that the second expansion of solutions to the infinity Laplace equation is independent of the geometry of the domain, quite different from the classical Laplacian. The second expansion obtained in this paper furthermore shows a substantial difference on the asymptotic behavior of boundary blow-up solutions between the infinity Laplacian and the classical Laplacian.

1. Introduction

In this paper we study the boundary asymptotic behavior of solutions to the infinity Laplace equation

$$\begin{cases} \Delta_\infty u = b(x)f(u), & x \in \Omega, \\ u = \infty, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded C^1 domain in $\mathbb{R}^N$ with $N \ge 2$, the weighted function b and the nonlinearity f satisfy

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(H-b) $b \in C(\bar{\Omega})$, $b > 0$ in Ω;
(H-f) $f \in W^{1,\infty}_{\text{loc}}[0, \infty)$, $f(0) = 0$, $f(s)/s$ is increasing in $(0, \infty)$.

The infinity Laplacian, defined as

$$\Delta_\infty u := \sum_{i,j=1}^{N} u_{x_i} u_{x_j} u_{x_i x_j} = \langle D^2 u(x) Du(x), Du(x) \rangle,$$

was proposed by Aronsson [4], and the infinity Laplace equation $\Delta_\infty u = 0$ just is the Euler–Lagrange equation for smooth absolute minimizers. Notice that the infinity Laplace equation is nonlinear and highly degenerate, and does not have smooth solutions in general. The equivalence of absolute minimizers and viscosity solutions (in the sense of [16]) of the Dirichlet problem to the infinity harmonic equation, as well as the uniqueness of solutions were proved by Jensen [25]. Refer to e.g. [8,9,15,19,26,30–33,37,39] and the survey [5] for important results of the infinity Laplace equations.

By a solution to the problem (1.1), we mean a nonnegative function $u \in C(\Omega)$ that satisfies the equation in the viscosity sense (see Section 2 for the definition) and the boundary condition with $u(x) \to \infty$ as the distance function $d(x) := \text{dist}(x, \partial\Omega) \to 0$. Such a solution is usually called a boundary blow-up solution or a large solution.

The boundary blow-up problems have been studied extensively in the context of the classical Laplace operator and other elliptic operators. Recently, the boundary blow-up problems have been extended to the elliptic problems involving the infinity Laplacian. Juutinen and Rossi [27] investigated the existence and uniqueness of solutions to the infinity Laplace problem

$$\begin{cases} \Delta_\infty^N u = u^q, & x \in \Omega, \\ u = \infty, & x \in \partial\Omega \end{cases} \tag{1.2}$$

with the normalized ∞-Laplacian

$$\Delta_\infty^N u := \frac{1}{|Du(x)|^2} \langle D^2 u(x) Du(x), Du(x) \rangle,$$

and proved that (1.2) admits a solution if and only if $q > 1$. They also obtained the boundary asymptotic estimates, and thus the uniqueness of solutions. The existence or nonexistence of boundary blow-up solutions to the problem

$$\begin{cases} \Delta_\infty u = h(x, u), & x \in \Omega, \\ u = \infty, & x \in \partial\Omega \end{cases} \tag{1.3}$$

with $h : \Omega \times [0, \infty) \to [0, \infty)$ continuous and nondecreasing in u for each $x \in \Omega$, was considered in [35]. In particular, it was shown that (1.3) with $h(x, u) = b(x) f(u)$ (i.e. the problem (1.1)) admits a nonnegative solution if and only if the Keller–Osserman condition

$$\int_1^\infty \frac{ds}{\sqrt[4]{F(s)}} < \infty, \quad F(s) = \int_0^s f(\tau) d\tau \tag{1.4}$$

holds, where a boundary asymptotic estimate was obtained as well with $b > 0$ on $\bar{\Omega}$ and some additional assumptions on Ω and b. The problem (1.3) for the case without the monotonicity restriction of h has been also studied, see [36].