J. Math. Anal. Appl. 436 (2016) 138–148

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Inversion of the elliptical Radon transform arising in migration imaging using the regular Radon transform

Sunghwan Moon [a,*], Joonghyeok Heo [b]

[a] *Department of Mathematical Sciences, Ulsan National Institute of Science and Technology (UNIST), Ulsan 44919, Republic of Korea*
[b] *UM Energy Institute, University of Michigan, Ann Arbor, MI 48109, USA*

ARTICLE INFO

Article history:
Received 14 December 2014
Available online 2 December 2015
Submitted by L. Fialkow

Keywords:
Radon transform
Tomography
Elliptical
Migration
Seismic reflection

ABSTRACT

In recent years, many types of elliptical Radon transforms that integrate functions over various sets of ellipses/ellipsoids have been considered, relating to studies in bistatic synthetic aperture radar, ultrasound reflection tomography, radio tomography, and migration imaging. In this article, we consider the transform that integrates a given function in $\mathbb{R}^n$ over a set of ellipses (when $n = 2$) or ellipsoids of rotation (when $n \geq 3$) with foci restricted to a hyperplane. We show a relation between this elliptical Radon transform and the regular Radon transform, and provide the inversion formula for the elliptical Radon transform using this relation. Numerical simulations are performed to demonstrate the suggested algorithms in two dimensions, and these simulations are also provided in this article.

1. Introduction

Radon-type transforms that assign to a given function its integrals over various sets of ellipses/ellipsoids arise in migration imaging under an assumption that the medium is acoustic and homogeneous. The aim of migration is to construct an image of the inside of the earth from seismic reflections recorded at its surface [9,25]. A graphical approach called classical migration was developed systematically by Hagedoorn [11]. Classical migration had been abandoned after the wave-equation method was introduced by Claerbout [5] in 1971. Gazdag and Sguazzero pointed out that the classical migration procedures that existed at that time were not based on a completely sound theory [9]. However, a correct construction for the wave-equation method was often difficult to find because the experiment data did not fit into a single wave equation. To adapt the diffraction stack to borehole seismic experiments, a new approach to seismic migration was found. This approach gave classical migration a sound theory. After that discovery, classical migration has attracted many researchers in the field. The underlying idea is that seismic data in the far field can be

* Corresponding author.
E-mail address: shmoon@unist.ac.kr (S. Moon).

http://dx.doi.org/10.1016/j.jmaa.2015.11.043
0022-247X/

regarded as if the data are coming from integrals of the earth's acoustic scattering potential over surfaces determined by the velocity model [16]. These Radon-type transforms relate to migration imaging as well as Bistatic Synthetic Aperture Radar (BiSAR) [2,6,13,14,32], Ultrasound Reflection Tomography (URT) [1,10,26], and radio tomography [27–29].

Because of these applications, there have been several papers devoted to the topic of elliptical Radon transforms. The family of ellipses with one focus fixed at the origin and the other one moving along a given line was considered in [13]. In the same paper, the family of ellipses with a fixed focal distance was also studied. The authors of [1,10] dealt with the case of circular acquisition, when the two foci of ellipses with a given focal distance are located on a given circle. A family of ellipses with two moving foci was also handled in [6]. Radio tomography, which uses a wireless network of radio transmitters and receivers to image the distribution of attenuation within the network, was discussed in [27–29]. They approximated the obtained signal by the volume integral of the attenuation over an ellipsoid with foci at these two devices. One work [17,18] derived two inversion formulas of this volume integral of the attenuation over this ellipsoid and studied its properties. Many works found an approximate inversion for elliptical Radon transforms.

Here we consider migration imaging and introduce a new type of an elliptical Radon transform obtained by restricting the position of the source and receiver in migration imaging. We find an explicit inversion formula for this elliptical Radon transform arising in migration imaging which is the line/area integral of the function over the ellipse/ellipsoid with foci restricted to a hyperplane.

The rest of this paper is organized as follows. The problem of interest is stated precisely and the elliptical Radon transform is formulated in section 2. In section 3, we show how to reduce the elliptical Radon transform to the regular Radon transform. The numerical simulation to demonstrate the suggested 2-dimensional algorithm is presented in section 4.

2. Formulation of the problem

Let $\mathbf{s} \in \mathbb{R}^3$ and $\mathbf{r} \in \mathbb{R}^3$ represent 3-dimensional source and receiver positions, respectively. For fixed points $(\mathbf{s}, \mathbf{r})$, an isochron surface $I_{(\mathbf{s},\mathbf{r},t)}$[1] is a surface consisting of image points $\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3$ associated by the travel time function $\tau(\mathbf{x}, \mathbf{y})$, which gives the travel time $t \in [0, \infty)$ between two points $\mathbf{x} \in \mathbb{R}^3$ and $\mathbf{y} \in \mathbb{R}^3$ with a known velocity $v(\mathbf{x})$ (see [16]).

Mathematically, $I_{(\mathbf{s},\mathbf{r},t)}$ can be described as the set of the image points $\mathbf{x}$ satisfying the constraint that the total travel time from the source $\mathbf{s}$ though the image point $\mathbf{x}$ to the receiver $\mathbf{r}$ is constant and equal to t. The isochron surface $I_{(\mathbf{s},\mathbf{r},t)}$ can be represented as

$$I_{(\mathbf{s},\mathbf{r},t)} = \{\mathbf{x} \in \mathbb{R}^3 : t = \tau(\mathbf{x}, \mathbf{s}) + \tau(\mathbf{x}, \mathbf{r})\}.$$

Seismic experiments yield data $g(\mathbf{s}, \mathbf{r}, t)$ which are functions of the source position $\mathbf{s}$, receiver position $\mathbf{r}$, and time t. Assuming an object function $f(\mathbf{x})$ on $\mathbb{R}^3$, the data g is modelled by the integral of f over $I_{(\mathbf{s},\mathbf{r},t)}$, i.e.,

$$g(\mathbf{s}, \mathbf{r}, t) = \int\limits_{I_{(\mathbf{s},\mathbf{r},t)}} f(\mathbf{x}) d\mathbf{x} \quad \text{(see [16])}.$$

The two detectors have nonzero sizes and time is also passing, so it is reasonable to assume that what we measure is the "average"-concentrated near the location of the detectors and nearly zero sufficiently far away from the detectors-over a small region of space and a small time interval preceding the time t. In mathematical terms, our data can be written as

[1] Hagedoorn called this set a surface of maximum concavity (see [11]).