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The second order expansion of boundary blow-up solutions for infinity-Laplacian equations

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ABSTRACT

In this paper, by using Karamata regular variation theory and a perturbed argument, we study the second order expansion of viscosity solutions to boundary blow-up elliptic problem $\Delta_\infty u = b(x)f(u)$, $x \in \Omega$, $u|_{\partial\Omega} = +\infty$, where Ω is a bounded domain with C^2-boundary in $\mathbb{R}^N$, $b \in \mathrm{C}(\bar{\Omega})$ is non-negative and non-trivial in Ω, $f \in C^1([0,\infty))$ is a normalized regularly varying function at infinity with index $\gamma > 3$.

1. Introduction and main results

Let Ω be a bounded domain with C^2-smooth boundary in $\mathbb{R}^N$. In this paper, we investigate the second order expansion of viscosity solutions to the following problem

$$\Delta_\infty u = b(x)f(u),\ x \in \Omega,\ u \geq 0,\ u|_{\partial\Omega} = +\infty, \tag{1.1}$$

where the last condition means that the solution u is blow-up on $\partial\Omega$, which is understood in the sense that

$$u(x) \to \infty \text{ as } d(x) := \mathrm{dist}(x, \partial\Omega) \to 0$$

and the operator Δ_∞ is given by

$$\Delta_\infty u := \sum_{i=1}^{N} D_i u D_j u D_{ij} u, \tag{1.2}$$

b satisfies

($\mathbf{b_1}$) $b \in C(\bar{\Omega})$, is non-negative and non-trivial in Ω;

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($\mathbf{b_2}$) there exist $k \in \Lambda$ and $B_0 \in \mathbb{R}^N$ such that

$$b(x) = k^4(d(x))\big(1 + B_0 d(x) + o(d(x))\big) \text{ near } \partial\Omega,$$

where Λ denotes the set of all positive non-decreasing functions in $C^1((0, \delta_0))$ which satisfy

$$\lim_{t\to 0^+} \frac{d}{dt}\left(\frac{K(t)}{k(t)}\right) := D_k \in [0,1],\ K(t) := \int_0^t k(s)ds,$$

and f satisfies

($\mathbf{f_1}$) $f \in C^1([0,\infty))$, $f(0) = 0$, f is increasing on $(0,\infty)$;
($\mathbf{f_2}$) there exist $\gamma > 3$ and a function $g \in C^1([S_0,\infty))$ for S_0 large enough such that

$$\frac{sf'(s)}{f(s)} := \gamma + g(s), s \geq S_0 \text{ with } \lim_{s\to\infty} g(s) = 0;$$

($\mathbf{f_3}$) there exists $\theta \geq 0$ such that

$$\lim_{s\to\infty} \frac{sg'(s)}{g(s)} = -\theta,$$

moreover, we need to further assume that

($\mathbf{f_4}$) if $\theta = 0$ in ($\mathbf{f_3}$), there exists $\sigma \in \mathbb{R}$ such that

$$\lim_{s\to\infty} (\ln s)^\beta g(s) = \sigma.$$

The operator (1.2) is so called ∞-Laplacian, which is first studied by Aronsson [4] motivated by the geometric problem of finding the so-called absolutely minimizing functions in Ω. In fact, this operator is a quasilinear and highly degenerate elliptic operator, and this degeneracy accounts for the non-existence, in general, of smooth solutions to Dirichlet problems. Therefore solutions are understood in the viscosity sense, a concept developed by Crandall, Lions [23] and Crandall, Evans and Lions [22], and to be defined in Section 4. Then, Jensen [30] showed that a necessary and sufficient condition for $u \in C(\bar{\Omega})$ being an absolute minimizing is that u solve the infinity harmonic equation $\Delta_\infty v = 0$ in the viscosity sense. In particular, the author proved the uniqueness of viscosity solutions. Since then, the infinity Laplacian equation has been discussed extensively by many authors in previous literatures. For further insight on the infinity Laplacian, please refer to [3,5,10–12,16,21,25,31–34,43,45,46,48,53,56] and the references therein.

Recently, many authors investigated the following more general inhomogeneous equation.

$$\Delta_\infty u = f(x,u), \tag{1.3}$$

where $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a continuous function. For the case that $f(x,u)$ is independent of u, Lu and Wang [40,41] first investigated the existence, uniqueness and stability of viscosity solutions to the Dirichlet problem of (1.3). For the case that the inhomogeneous $f(x,u)$ depends on both the variables x and u, Bhattacharya and Mohammed [13,14] analyzed the existence and nonexistence of viscosity solutions to the Dirichlet problem of (1.3). Especially, authors [14] removed the sign and the monotonicity restrictions for the right hand side $f(x,u)$ which is added in [13]. For the case that $f(x,u) = b(x)f(u)$ with $b > 0$