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Asymptotic analysis for two joined thin slanting ferromagnetic films

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ABSTRACT

Starting from a $3D$ non-convex and nonlocal micromagnetic energy for ferromagnetic materials, we determine, via an asymptotic analysis, the free energy of two joined ferromagnetic thin films. We distinguish different regimes depending on the limit of the ratio between the small thickness of the two films.

1. Introduction

In this paper, starting from the classical $3D$ micromagnetic energy (cf. L.D. Landau and E.M. Lifshitz [23]), we determine via an asymptotic analysis, the free energy of a ferromagnetic multi-structure, composed of two joined films, forming an angle $\theta_0 \in]0,\pi[$ at the junction points $(h_n^a, x_2, 0)$ (see Fig. 1). Precisely, let $\Omega_n^{\theta_0} = \Omega_n^{a,\theta_0} \cup \Omega_n^b$, with $n \in \mathbb{N}$, $\Omega_n^b =]-\frac{1}{2}, \frac{1}{2}[^2 \times]-h_n^b, 0[$ and Ω_n^{a,θ_0} is a bounded domain in $\mathbb{R}^3$ with height 1, so that form an angle θ_0 with the domain Ω_n^b such that, for all $n \in \mathbb{N}$

$$\Omega_n^{a,\theta_0} = \Big\{(x_1,x_2,x_3) \in \mathbb{R}^3 : |x_1 - \cot\theta_0 x_3| < \frac{h_n^a}{2}, (x_2,x_3) \in]-\frac{1}{2}, \frac{1}{2}[\times [0, \sin\theta_0[\Big\},$$

where h_n^a and h_n^b are the small thickness of Ω_n^{a,θ_0} and Ω_n^b respectively, such that

$$\begin{cases} \lim_n h_n^a = 0 = \lim_n h_n^b, \\ \lim_n \dfrac{h_n^b}{h_n^a} = q \in [0,+\infty]. \end{cases} \tag{1.1}$$

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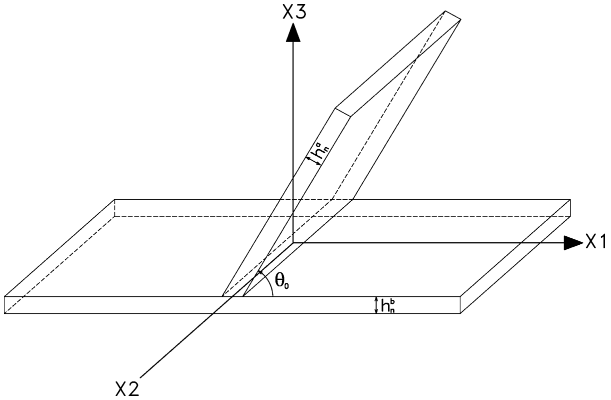

Fig. 1. $\Omega_n^{\theta_0}$.

In general, this type of thin films is used in apparatus for the storage of numerical information, such as the hard disks, magnetoresistive sensors, inductive thin films heads in magnetic, etc. The aim of this paper, is to study the asymptotic behavior as n diverges of the following problem:

$$\min\left\{ \int_{\Omega_n^{\theta_0}} (\alpha|D\underline{m}|^2 + \varphi(\underline{m}) + \frac{1}{2}D\underline{\zeta}\underline{m} - 2F_n\underline{m})dx, \underline{m} \in H^1(\Omega_n^{\theta_0}, S^2) \right\} \tag{1.2}$$

subject to

$$div(-D\underline{\zeta} + \underline{m}) = 0 \qquad \text{in } \mathbb{R}^3 \tag{1.3}$$

where α is the exchange constant, $\varphi : S^2 \to [0, +\infty[$ is a continuous and even function, $F_n \in L^2(\Omega_n^{\theta_0}, \mathbb{R}^3)$, and S^2 denotes the unit sphere of $\mathbb{R}^3$. In classical theory of micromagnetics, $\underline{m} : \Omega_n^{\theta_0} \longrightarrow \mathbb{R}^3$ denotes the magnetization and the body is always locally magnetized to a saturation magnetization $|\underline{m}(x)| = m_s(T) > 0$ unless the local temperature T is greater or equal to Curie temperature depending on the body, in the latter case $\underline{m}(T) = 0$. It is understood that $\underline{m}(x) = 0$ in $\mathbb{R}^3 \setminus \Omega_n^{\theta_0}$. This model was proposed by Brown in [6].

We suppose that the temperature is a constant and lower than Curie temperature and, without loss of generality, we assume that $|\underline{m}| = 1$, i.e. $\underline{m}(x) \in S^2$ a.e. in $\Omega_n^{\theta_0}$. The function $\underline{\zeta}$: $\mathbb{R}^3 \to \mathbb{R}$ denotes the magnetic field potential. The magnetic field potential and the magnetization $\underline{m}$ are connected by equation (1.3). To reformulate the problem on a fixed domain $\Omega^{\theta_0} = \Omega^{a,\theta_0} \cup \Omega^b$, where $\Omega^b =]-\frac{1}{2}, \frac{1}{2}[^2 \times]-1, 0[$ and Ω^{a,θ_0} is the rescaled domain of Ω_n^{a,θ_0}, we derive the limit problem which depends on the limit $\lim_n \frac{h_n^b}{h_n^a} = q \in [0, +\infty]$. Precisely, if $q \in]0, +\infty[$, we prove that the limit problem is given by

$$\begin{aligned}
&\min\left\{ \int_{\Omega^{a,\theta_0}} \left(\alpha|(D_{x_1}\xi^a, D_{x_2}\mu^a, D_{x_3}\mu^a - \cot\theta_0 D_{x_1}\xi^a)|^2 + \varphi(\mu^a) + \frac{1}{2}|\sin\theta_0\mu_1^a - \cos\theta_0\mu_3^a|^2\right)dx \right.\\
&-2\int_{\Omega^{a,\theta_0}} f^a(x_1,x_2,x_3)\mu^a dx - 2q\int_{\Omega^b} f^b(x_1,x_2,x_3)\mu^b dx +\\
&q\int_{]-\frac{1}{2},\frac{1}{2}[^2} \left(\alpha|(D_{x_1}\mu^b, D_{x_2}\mu^b)|^2 + \varphi(\mu^b) + \frac{1}{2}|\mu_3^b|^2\right)dx_1dx_2,\\
&(\mu^a, \mu^b, \xi^a) \in H^1(\Omega^{a,\theta_0}, S^2) \times H^1(\Omega^b, S^2) \times \mathcal{F},\\
&\left.\mu^a(x_2, 0) = \mu^b(0, x_2) \text{ in }]-\frac{1}{2}, \frac{1}{2}[\right\},
\end{aligned} \tag{1.4}$$