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Radial positive solutions of nonlinear elliptic systems with Neumann boundary conditions

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ABSTRACT

We consider radial positive solutions of elliptic systems of the form

$$\begin{cases} -\Delta u + u = \alpha(|x|)f(u,v) & \text{in } B_R, \\ -\Delta v + v = \beta(|x|)g(u,v) & \text{in } B_R, \\ \partial_\nu u = \partial_\nu v = 0 & \text{on } \partial B_R, \end{cases}$$

where essentially α, β are assumed to be radially nondecreasing weights and f, g are nondecreasing in each component. We show the existence of at least one nondecreasing nontrivial radial solutions. Our result is sharp and is a complement for one of the main results in Bonheure et al. (2013) [2]. The proof of our main result is based upon bifurcation techniques.

1. Introduction

Let B_R be the ball of radius R in $\mathbb{R}^N$, with $N \geq 2$. We consider the Neumann problem

$$\begin{cases} -\Delta u + u = \alpha(|x|)f(u,v) & \text{in } B_R, \\ -\Delta v + v = \beta(|x|)g(u,v) & \text{in } B_R, \\ \partial_\nu u = \partial_\nu v = 0 & \text{on } \partial B_R, \end{cases} \tag{1.1}$$

where α, β and f, g satisfy the assumptions

(A) $\alpha, \beta \in L^1(0,R)$ are nonnegative, nondecreasing and not identically zero;
(H1) $f, g \in C(\mathbb{R}^+ \times \mathbb{R}^+)$ are nonnegative and nondecreasing in each variable.

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Existence of positive radial solutions of elliptic equations imposing Neumann conditions has been widely considered in the literature. For example, see [1,2,8] and the references therein. It is worth remarking that the key point in [8] consists in the observation that restricting the set of trial functions to nonnegative and nondecreasing radial functions in $H^1(B_R)$ which gives rise to boundedness and compactness properties even for supercritically growing nonlinearities. This idea is exploited in a topological arguments in [1] and further developed for elliptic system (1.1) in [2]. Bonheure et al. proved the following:

Theorem A. *(See [2, Theorem 1.1].) Under assumptions (A), (H1) and*

$$\lim_{s+t\to 0^+}\frac{f(s,t)+g(s,t)}{s+t}=0,\qquad \lim_{s+t\to\infty}\frac{f(s,t)+g(s,t)}{s+t}=\infty,$$

problem (1.1) admits at least one nontrivial solution (u,v) with u and v both nonnegative and nondecreasing.

Bonheure et al. used an approach based on topological fixed point theory and invariance properties of the cone of nonnegative, nondecreasing radial functions in $H^1(B_R)$. However, they give no information on the global structure of positive radial solutions of (1.1), and they do not deal with the case

$$\lim_{s+t\to 0^+}\frac{f(s,t)+g(s,t)}{s+t}>0,\qquad \lim_{s+t\to\infty}\frac{f(s,t)+g(s,t)}{s+t}<\infty. \tag{1.2}$$

It is the purpose of the present paper is to study the global structure of positive radial solutions of (1.1) under (1.2) and give a complement of the results of Theorem A to a new class of functions f and g by means of a new approach based on global bifurcation theorem of Dancer [4]. We shall provide sharp conditions which guarantee the existence of nonnegative, nondecreasing radial solution of (1.1), see Theorem 3.1 below.

The paper is organized as follows. In Section 2 we introduce some functional setting and preliminary results. Finally in Section 3 we state and prove our main results on the existence of nonnegative, nondecreasing radial solutions in $H^1(B_R)$ by applying the well-known global bifurcation theorem due to Dancer [4].

2. Functional setting and preliminary properties

Suppose that E is a real Banach space with norm $||\cdot||$. If B is a continuous linear operator on E, define $r(B)$ to be the spectral radius of the natural extension of B to the complexification of E. Finally, let E^* denote the dual of E.

Let K be a cone in E. Then K is closed. If $x,y\in E$, $x>y$ means that $x-y\in K$. Define

$$E_K=\{x-y: x,y\in K\},\quad K_\rho=\{x\in K: ||x||<\rho\}.$$

E_K and $\bar{E}_K$ are subspaces of E.

A linear operator B on E_K is said to be *positive* if $B(K)\subset K$. If $B^{[1]}$ and $B^{[2]}$ are both linear operators on E_K, $B^{[1]}\geq B^{[2]}$ means that $B^{[1]}-B^{[2]}$ is positive. A linear operator B on E_K is said to be *K-continuous* if the mapping $B|_K$ is continuous. In this case, an elementary argument shows that $\sup\{||Bx||: x\in K_1\}$ is finite. Let $||B||_K$ denote this supremum. A linear operator B on E_K is defined to be *K-completely continuous* if $B(\bar{K}_1)$ is precompact and B is *K-continuous*. Finally, if B is a K-continuous positive linear operator on E_K, define

$$r_K(B)=\lim_{n\to\infty}(||B^n||_K)^{1/n}. \tag{2.1}$$

Bonsall [3, Chapter 5] proves that this limit exists. Note that $r_K(B)\leq ||B||_K<\infty$. If B extends to a continuous linear operator on E, $r_K(B)\leq r(B)$.