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Existence of positive radial solutions for superlinear, semipositone problems on the exterior of a ball

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ABSTRACT

We study positive radial solutions to $-\Delta u = \lambda K(|x|)f(u)$; $x \in \Omega_e$ where $\lambda > 0$ is a parameter, $\Omega_e = \{x \in \mathbb{R}^N \mid |x| > r_0, r_0 > 0, N > 2\}$, Δ is the Laplacian operator, $K \in C([r_0, \infty), (0, \infty))$ satisfies $K(r) \leq \frac{1}{r^{N+\mu}}$; $\mu > 0$ for $r >> 1$, and $f \in C^1([0, \infty), \mathbb{R})$ is a class of non-decreasing functions satisfying $\lim_{s\to\infty} \frac{f(s)}{s} = \infty$ (superlinear) and $f(0) < 0$ (semipositone). We consider solutions, u, such that $u \to 0$ as $|x| \to \infty$, and which also satisfy the nonlinear boundary condition $\frac{\partial u}{\partial \eta} + \tilde{c}(u)u = 0$ when $|x| = r_0$, where $\frac{\partial}{\partial \eta}$ is the outward normal derivative, and $\tilde{c} \in C([0, \infty), (0, \infty))$. We will establish the existence of a positive radial solution for small values of the parameter λ. We also establish a similar result for the case when u satisfies the Dirichlet boundary condition ($u = 0$) for $|x| = r_0$. We establish our results via variational methods, namely using the Mountain Pass Lemma.
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1. Introduction

We study positive, radial solutions to steady state reaction diffusion equations of the forms

$$\begin{cases} -\Delta u = \lambda K(|x|)f(u); & x \in \Omega_e \\ u = 0; & |x| = r_0 \\ u \to 0; & |x| \to \infty \end{cases} \tag{1}$$

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and

$$\begin{cases} -\Delta u = \lambda K(|x|)f(u); & x \in \Omega_e \\ \frac{\partial u}{\partial \eta} + \tilde{c}(u)u = 0; & |x| = r_0 \\ u \to 0; & |x| \to \infty, \end{cases} \tag{2}$$

where $\lambda > 0$ is a parameter, $\Omega_e = \{x \in \mathbb{R}^N \mid |x| > r_0, r_0 > 0, N > 2\}$, Δ is the Laplacian operator, $K \in C([r_0, \infty), (0, \infty))$ satisfies $K(r) \leq \frac{1}{r^{N+\mu}}$; $\mu > 0$ for $r >> 1$, $\frac{\partial}{\partial \eta}$ is the outward normal derivative, and $\tilde{c} \in C([0, \infty), (0, \infty))$. Here, the reaction term $f : [0, \infty) \to \mathbb{R}$ is a nondecreasing, C^1 function such that

(F1) $\lim_{s\to\infty} \frac{f(s)}{s} = \infty$ (superlinear at ∞), and
(F2) $f(0) < 0$ (semipositone).

The case when $f(0) < 0$ is referred to in the literature as a semipositone problem, and it has been well documented (see [22,6]) that they pose considerably more challenges in the study of positive solutions than the case where $f(0) > 0$ (positone problems). For a rich history of superlinear, semipositone problems on bounded domains with Dirichlet boundary conditions, see [2–5,7,8,10–16,20,21,26]. The main focus of this paper is to extend an important existence result for $\lambda \approx 0$ obtained in the case of bounded domains with Dirichlet boundary conditions to a domain exterior to a ball, and also to problems involving classes of nonlinear boundary conditions on the boundary of the ball. Recently, the authors in [1] studied (1) via degree theory arguments. In this paper we establish existence of positive radial solutions for (1) as well as (2) via variational methods.

The nonlinear boundary conditions discussed above occur very naturally in applications. See [19,23,25] for a detailed description. Here, the authors study a model arising in combustion theory involving a sublinear reaction term. See also [9] for an extension of this work to an exterior domain.

Note that the change of variables $r = |x|$ and $S = \left(\frac{r}{r_0}\right)^{2-N}$ transforms (1), (2) respectively into the following boundary value problems (see Appendix 8.1 in [9] for details):

$$\begin{cases} -u'' = \lambda h(x)f(u); & x \in (0,1) \\ u(0) = 0 = u(1) \end{cases} \tag{3}$$

and

$$\begin{cases} -u'' = \lambda h(x)f(u); & x \in (0,1) \\ u(0) = 0 \\ u'(1) + c(u(1)) \cdot u(1) = 0 \end{cases} \tag{4}$$

where $h(t) = \frac{r_0^2}{(2-N)^2} t^{\frac{-2(N-1)}{N-2}} K\left(r_0 t^{\frac{1}{2-N}}\right)$ and $c(s) = \frac{r_0}{N-2}\tilde{c}(s)$. We will only assume $K(r) \leq \frac{1}{r^{N+\mu}}$ for $r >> 1$ and for some $\mu \in (0, N-2)$. Then $h \in C((0,1], (0,\infty))$ could be singular at $t = 0$. (If $\mu \geq N-2$, h will be non-singular at $t = 0$ and will therefore be an easier case to study.) Note that h is still an $L^1(0,1)$ function with $h(t) > 0$ $\forall t \in (0,1]$.

We will study positive solutions of (3) and (4) in the space $C^2(0,1) \cap C^1[0,1]$ when f satisfies the additional conditions

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