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Asymptotic formula on average path length of fractal networks modeled on Sierpinski gasket ☆

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ABSTRACT

In this paper, we introduce a new method to construct evolving networks based on the construction of the Sierpinski gasket. Using self-similarity and renewal theorem, we obtain the asymptotic formula for average path length of our evolving networks.
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1. Introduction

The Sierpinski gasket described in 1915 by W. Sierpiński is a classical fractal. Suppose K is the solid regular triangle with vertexes $a_1 = (0, 0)$, $a_2 = (1, 0)$, $a_3 = (1/2, \sqrt{3}/2)$ (see Fig. 1). Let $T_i(x) = x/2 + a_i/2$ be the contracting similitude for $i = 1, 2, 3$. Then $T_i : K \to K$ and the Sierpinski gasket E is the self-similar set, which is the unique invariant set [7] of IFS $\{T_1, T_2, T_3\}$, satisfying

$$E = \cup_{i=1}^{3} T_i(E).$$

The Sierpinski gasket is important for the study of fractals, e.g., the Sierpinski gasket is a typical example of post-critically finite self-similar fractals on which the Dirichlet forms and Laplacians can be constructed by Kigami [8,9], see also Strichartz [12].

For the word $\sigma = i_1 \cdots i_k$ with letters in $\{1, 2, 3\}$, i.e., every letter $i_t \in \{1, 2, 3\}$ for all $t \le k$, we denote by $|\sigma|(= k)$ the length of word σ. Given words $\sigma = i_1 \cdots i_k$ and $\tau = j_1 \cdots j_n$, we call σ a prefix of τ and

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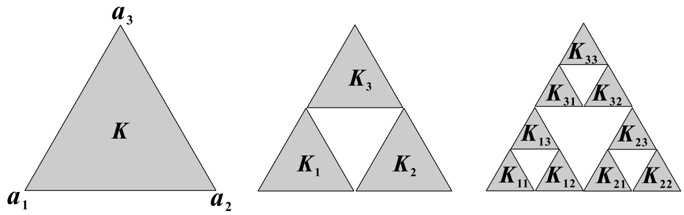

Fig. 1. The first two constructions of Sierpinski gasket.

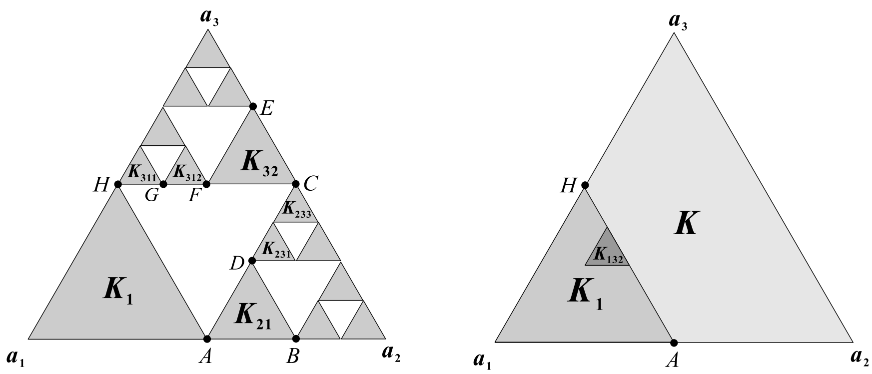

Fig. 2. Geodesic paths.

denote by $\tau \prec \sigma$, if $k < n$ and $i_1 \cdots i_k = j_1 \cdots j_k$. We also write $\tau \preceq \sigma$ if $\tau = \sigma$ or $\tau \prec \sigma$. When $\tau \prec \sigma$ with $|\tau| = |\sigma| - 1$, we say that τ is the father of σ and σ is a child of τ. Given $\sigma = i_1 \cdots i_k$, we write $T_\sigma = T_{i_1} \circ \cdots \circ T_{i_k}$ and $K_\sigma = T_\sigma(K)$ which is a solid regular triangle with side length $2^{-|\sigma|}$. For notational convenience, we write $K_\emptyset = K$ with empty word $\emptyset$. We also denote $|\emptyset| = 0$. If $\tau \prec \sigma$, then $K_\sigma \subset K_\tau$. For solid triangle K_σ with word σ, we denote by ∂K_σ its boundary consisting of 3 sides, where every side is a line segment with side length $2^{-|\sigma|}$.

Complex networks arise from natural and social phenomena, such as the Internet, the collaborations in research, and the social relationships. These networks have in common two structural characteristics: the small-world effect and the scale-freeness (*power-law* degree distribution), as indicated, respectively, in the seminal papers by Watts and Strogatz [14] and by Barabási and Albert [1]. In fact complex networks also exhibit *self-similarity* as demonstrated by Song, Havlin and Makse [11] and fractals possess the feature of *power law* in terms of their fractal dimension (e.g. see [4]). Recently self-similar fractals are used to model evolving networks, for example, in a series of papers, Zhang et al. [16,17,6] use the Sierpinski gasket to construct evolving networks. There are also some complex networks modeled on self-similar fractals, for example, Liu and Kong [10] and Chen et al. [2] study Koch networks, Zhang et al. [15] investigate the networks constructed from Vicsek fractals. See also Dai and Liu [3], Sun et al. [13] and Zhou et al. [18].

In the paper, we introduce a new method to construct evolving networks modeled on Sierpinski gasket and study the asymptotic formula for average path length.

Since E is connected, we can construct the network from geometry as follows.

Fix an integer t, we consider a network G_t with vertex set $V_t = \{\sigma : 0 \le |\sigma| \le t\}$ where $\#V_t = 1 + 3 + \ldots + 3^t = \frac{1}{2}(3^{t+1} - 1)$. For the edge set of G_t, there is a unique edge between distinct words σ and τ (denoted by $\tau \sim \sigma$) if and only if

$$\partial K_\sigma \cap \partial K_\tau \ne \varnothing. \tag{1.1}$$

We can illustrate the geodesic paths in Fig. 2 for $t = 3$. We have $233 \sim 32 \sim 312$ since $\partial K_{233} \cap \partial K_{32} = \{C\}$ and $\partial K_{32} \cap \partial K_{312} = \{F\}$. We also get another geodesic path from 233 to 312: $233 \sim 3 \sim 312$ since

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