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# On the existence and stability of minimizers in ferromagnetic nanowires

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ABSTRACT

We study static 180 degree domain walls in infinite magnetic wires with bounded, $C^1$ and rotationally symmetric cross sections. We prove an existence of global minimizers for the energy of micromagnetics for any bounded $C^1$ cross sections. Under some asymmetry of cross sections we prove a stability result for the minimizers, namely, we show that vectors of micromagnetics having an energy close to the minimal one, must be $H^1$ close to the actual minimizer, which is itself close to the minimizer of the limit energy up to a rotation and a translation.



## 1. Introduction

In the theory of micromagnetics to any domain $\Omega \in \mathbb{R}^3$ and a unit vector field (called magnetization) $m \colon \Omega \to \mathbb{S}^2$ with $m = 0$ in $\mathbb{R}^3 \setminus \Omega$ the energy of micromagnetics is assigned:

$$E(m) = A_{ex} \int\limits_{\Omega} |\nabla m|^2 + K_d \int\limits_{\mathbb{R}^3} |\nabla u|^2 + Q \int\limits_{\Omega} \varphi(m) - 2 \int\limits_{\Omega} H_{ext} \cdot m,$$

where $A_{ex}$, $K_d$, $Q$ are material parameters, $H_{ext}$ is the externally applied magnetic field, $\varphi$ is the anisotropy energy density and $u$ is obtained from Maxwell's equations of magnetostatics,

$$\begin{cases} \text{curl } H_{ind} = 0 & \text{in} \quad \mathbb{R}^3 \\ \text{div}(H_{ind} + m) = 0 & \text{in} \quad \mathbb{R}^3, \end{cases}$$

i.e., $u$ is a weak solution of

$$\triangle u = \text{div } m \qquad \text{in} \quad \mathbb{R}^3.$$

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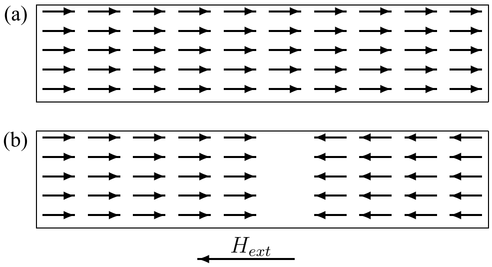


**Fig. 1.** (a) Homogeneous magnetization. (b) 1800 degree domain wall.

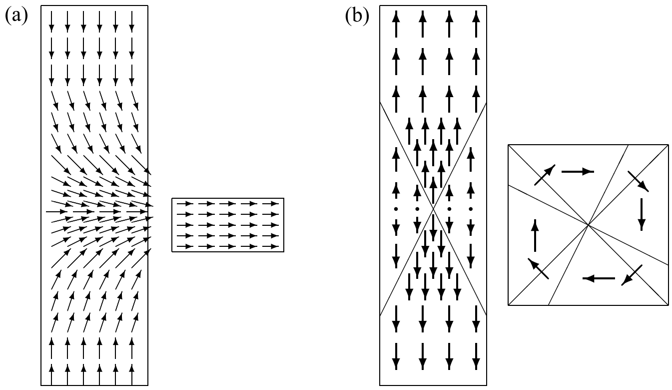


**Fig. 2.** (a) The transverse wall. (b) The vortex wall.

According to micromagnetics, stable magnetization patterns are described by the minimizers of the micromagnetic energy functional, see [8,9,16]. The study of magnetic wires and thin films has attracted significant attention in the recent years, see [1,2,7,10,15,19,21,22,24–26,28] for wires and [6,8,9,12,11,17,18, 20] for thin films. It has been suggested in [1] that magnetic nanowires can be effectively used as storage devices. When a homogeneous external field is applied in the axial direction of a magnetic wire facing the homogeneous magnetization direction (see Fig. 1), then at a critical strength of the field the reversal of the magnetization typically starts at one end of the wire creating a domain wall which starts moving along the wire. The domain wall separates the reversed and the not yet reversed parts of the wire (see Fig. 1). It is known that the magnetization pattern reversal time is closely related to the writing and reading speed of such a device, thus it is crucial to understand the magnetization reversal and switching processes. Several authors have numerically, experimentally and analytically observed two different magnetization modes in magnetic nanowires [10,15,13,19]. In [10] the magnetization reversal process has been studied numerically in cobalt nanowires by the Landau–Lifshitz–Gilbert equation. Two different domain wall types were observed. For thin cobalt wires with 10 nm in diameter the transverse mode has been observed: the magnetization is constant on each cross section and is moving along the wire. For thick wires, with diameters bigger that 20 nm the vortex wall has been observed: the magnetization is approximately tangential to the boundary and forms a vortex which propagates along the wire. In [15] the magnetization reversal process has been studied both numerically and experimentally. By considering a conical type wire so that the diameter of the cross section increases very slowly, the magnetization switching from the vortex wall to the transverse at a critical diameter has been observed, as the domain wall moves along the wire. The results in [10] and [15] were the same: in thin wires the transverse wall occurs, while in thick wires the vortex wall occurs.

In Fig. 2 one can see the transverse and the vortex wall longitudinal and cross section pictures for wires with a rectangular cross section.

In [14] the author studies a similar problem for thin films and derives a Γ-convergence result for the energies. In a series of papers the authors study the magnetization reversal process in thin films, identifying