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Lorentz hypersurfaces in $\mathbb{E}_1^{n+1}$ satisfying $\Delta\overrightarrow{H} = \lambda\overrightarrow{H}$ with at most three distinct principal curvatures

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A B S T R A C T

This paper is a continuation of our paper (Liu and Yang, 2014 [3]), where we investigate hypersurface M_r^n of pseudo-Euclidean space $\mathbb{E}_s^{n+1}$ satisfying $\Delta\overrightarrow{H} = \lambda\overrightarrow{H}$, and show that if M_r^n has diagonalizable shape operator with at most three distinct principal curvatures, then it has constant mean curvature. In this paper, we prove that the same conclusion remains true for such Lorentz hypersurfaces with non-diagonalizable shape operators.

1. Introduction

Let $x : M_r^n \to \mathbb{E}_s^{n+1}$ be an isometric immersion of a pseudo-Riemannian hypersurface M_r^n into a pseudo-Euclidean space $\mathbb{E}_s^{n+1}$. Denote by $\overrightarrow{H}$ and Δ the mean curvature vector field and the Laplace operator of M_r^n with respect to the induced metric.

If the hypersurface M_r^n satisfies the equation

$$\Delta\overrightarrow{H} = \lambda\overrightarrow{H}$$

for some real constant λ, then it is said to have *proper mean curvature vector field*. This equation is a natural generalization of the biharmonic submanifold equation $\Delta\overrightarrow{H} = 0$.

Under the assumption that the shape operator of M_r^n is diagonalizable, we proved in [3] that the hypersurface M_r^n with proper mean curvature vector field and at most three distinct principal curvatures has constant mean curvature.

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It is known from [4,5] that, for Lorentz hypersurface, the shape operator has three possible non-diagonalizable forms except for diagonalizable ones. So, in this paper, we show that the same conclusion in [3] remains true for these three cases and prove the following result.

Main Theorem. *Let M_1^n $(n \geq 4)$ be a nondegenerate Lorentz hypersurface with proper mean curvature vector field in $(n+1)$-dimensional pseudo-Euclidean space $\mathbb{E}_1^{n+1}$. Suppose that M_1^n has at most three distinct principal curvatures, then it has constant mean curvature.*

When $n = 3$, it is true automatically that M_1^3 has at most three distinct principal curvatures. In this case, the result has been proved by A. Arvanitoyeorgos et al. in [1, Theorem].

2. Preliminaries

2.1. Lorentz hypersurface in $\mathbb{E}_1^{n+1}$

A non-zero vector X in $\mathbb{E}_1^{n+1}$ is called *time-like*, *space-like* or *light-like*, according to whether $\langle X, X\rangle$ is negative, positive or zero.

Let M_1^n be a nondegenerate Lorentzian hypersurface in $\mathbb{E}_1^{n+1}$, $\overrightarrow{\xi}$ denote a unit normal vector field to M_1^n, then $\langle \overrightarrow{\xi}, \overrightarrow{\xi} \rangle = 1$, i.e. the normal vector to M_1^n is space-like.

Denote by ∇ and $\widetilde{\nabla}$ the Levi-Civita connections of M_1^n and $\mathbb{E}_1^{n+1}$ respectively. For any vector fields X, Y tangent to M_1^n, the Gauss formula is given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + h(X,Y)\overrightarrow{\xi},$$

where h is the scalar-valued second fundamental form. If we denote by A the shape operator of M_1^n associated with $\overrightarrow{\xi}$, then the Weingarten formula is given by

$$\widetilde{\nabla}_X \overrightarrow{\xi} = -A(X),$$

where $\langle A(X), Y\rangle = h(X,Y)$. The mean curvature vector $\overrightarrow{H} = H\overrightarrow{\xi}$ with $H = \frac{1}{n}\operatorname{tr} A$, determines a well-defined normal vector field to M_1^n in $\mathbb{E}_1^{n+1}$. The Codazzi and Gauss equations are given by (cf. [6])

$$(\nabla_X A)Y = (\nabla_Y A)X, \tag{1}$$

$$R(X,Y)Z = \langle A(Y), Z\rangle A(X) - \langle A(X), Z\rangle A(Y), \tag{2}$$

where

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.$$

According to [2], a hypersurface M_1^n of $\mathbb{E}_1^{n+1}$ is said to have proper mean curvature vector field, if and only if the following two equations hold:

$$A(\nabla H) = -\frac{n}{2} H(\nabla H), \tag{3}$$

$$\Delta H + H \operatorname{tr} A^2 = \lambda H, \tag{4}$$

where the Laplace operator Δ acting on scalar-valued function f is given by