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Finite-dimensional global attractor of the Cahn–Hilliard–Brinkman system

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ABSTRACT

Our aim in this paper is to study the asymptotic behavior of the Cahn–Hilliard–Brinkman system. We obtain the existence of a finite fractal dimensional global attractor $\mathcal{A}$ in $H^4(\Omega)\cap V_I$ by asymptotical a priori estimates for the Cahn–Hilliard–Brinkman system.

1. Introduction

This paper is concerned with the following Cahn–Hilliard–Brinkman system:

$$\frac{\partial\phi}{\partial t} + u\cdot\nabla\phi = \nabla\cdot(M\nabla\mu),\ \ (x,t)\in\Omega\times\mathbb{R}^+, \tag{1.1}$$

$$\mu = -\epsilon\Delta\phi + \frac{1}{\epsilon}f(\phi),\ \ (x,t)\in\Omega\times\mathbb{R}^+, \tag{1.2}$$

$$-\nu\Delta u + \eta u = -\nabla p - \gamma\phi\nabla\mu,\ \ (x,t)\in\Omega\times\mathbb{R}^+, \tag{1.3}$$

$$\nabla\cdot u = 0,\ \ (x,t)\in\Omega\times\mathbb{R}^+, \tag{1.4}$$

where $\Omega\subset\mathbb{R}^3$ is a bounded domain with smooth boundary $\partial\Omega$, $\mathbb{R}^+=[0,+\infty)$, $\nu>0$ is the viscosity, $\eta>0$ is the fluid permeability, p is the fluid pressure, $M>0$ stands for the mobility, $\epsilon>0$ is related to the diffuse interface thickness, f is the derivative of a double well potential $F(s)=\frac{1}{4}(s^2-1)^2$ describing phase separation, and $\gamma>0$ is a surface tension parameter.

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Equations (1.1)–(1.4) are subject to the following boundary conditions

$$u(x,t)=0,\ \ (x,t)\in\partial\Omega\times\mathbb{R}^+, \tag{1.5}$$

$$\frac{\partial\phi}{\partial\vec{n}}=\frac{\partial\mu}{\partial\vec{n}}=0,\ \ (x,t)\in\partial\Omega\times\mathbb{R}^+ \tag{1.6}$$

and initial condition

$$\phi(x,0)=\phi_0(x), \tag{1.7}$$

where $\vec{n}$ is the normal vector on $\partial\Omega$.

A diffuse interface variant of Brinkman equation has been proposed to model phase separation of incompressible binary fluids in a porous medium (see [14]). The coupled system consists of a convective Cahn–Hilliard equation for the phase field ϕ, i.e., the difference of the relative concentrations of the two phases, and a modified Darcy equation proposed by H.C. Brinkman [5] in 1947 for the fluid velocity u. This equation incorporates a diffuse interface surface force proportional to $\phi\nabla\mu$, where μ is the so-called chemical potential which is the variational derivative of the free energy functional

$$E(\phi)=\int\limits_\Omega \frac{\epsilon}{2}|\nabla\phi|^2+\frac{1}{\epsilon}F(\phi)\,dx.$$

For this reason, equations (1.1)–(1.4) have been called Cahn–Hilliard–Brinkman system. Such a system belongs to a class of diffuse interface models which are used to describe the behavior of multi-phase fluids. The Cahn–Hilliard–Navier–Stokes system has been investigated from the numerical and analytical viewpoint in several papers (see, e.g., [1,2,4,6,7,9–11,13,16,19,21]). The long-time behavior and well-posedness of solutions for the Cahn–Hilliard–Hele–Shaw system were proved in [17,18]. In [12], the authors have considered the well-posedness and long-time behavior of solutions for a non-autonomous Cahn–Hilliard–Darcy system with mass source modeling tumor growth which is more complicated than the Cahn–Hilliard–Brinkman system thanks to the compressibility of the fluid. Meanwhile, they established the existence of a pullback attractor in $H^2(\Omega)$, and proved any global weak/strong solution is convergent to a single steady state as time $t\to+\infty$ and obtained its convergence rate.

The Cahn–Hilliard–Brinkman system (1.1)–(1.4) with M, ν, and η possibly depending on ϕ has been analyzed from the numerical viewpoint in [7,8]. From the analytical point of view, the authors in [3] have considered the well-posedness of solutions, the existence of a global attractor in $H^1(\Omega)$ for the Cahn–Hilliard–Brinkman system (1.1)–(1.4) with positive constants M, ν, η, ϵ and more general $f(u)$, established the convergence of a given weak solution to a single equilibrium via the Lojasiewicz–Simon inequality and gave its convergence rate. Furthermore, the authors have also studied the behavior of the solutions as the viscosity goes to zero, i.e., the existence of a weak solution to the Cahn–Hilliard–Hele–Shaw system was proved as the limit of solutions to the Cahn–Hilliard–Brinkman system when the Cahn–Hilliard–Brinkman system approaches the Cahn–Hilliard–Hele–Shaw system.

It is well-known that the asymptotical behavior of solutions of a dissipative evolution equation can be adequately described in terms of its global attractor $\mathcal{A}$. In many problems, the influence of initial data has vanished after a long time has elapsed, therefore permanent regimes are of importance. The simplest permanent regimes are described by time-independent functions that are solutions of the corresponding elliptic equation. Such regimes are important but very special and it is included in the global attractor $\mathcal{A}$, which implies that the regularity of solutions of elliptic equation can be obtained from the regularity of global attractor of the corresponding evolution equation if its global attractor exists. From the result of [3], we know the fact that each trajectory $\phi(t)$ of the Cahn–Hilliard–Brinkman system originating from the initial data ϕ_0 will converge to a stationary solution w of the elliptic equation