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An optimal consumption, investment and voluntary retirement choice problem with disutility and subsistence consumption constraints: A dynamic programming approach

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ABSTRACT

In this paper, we investigate an optimal consumption/portfolio problem of an agent with voluntary retirement and subsistence consumption constraints before retirement. We assume that the agent's utility function of consumption is of CRRA type and the agent suffers a utility loss from labor before retirement. We use the dynamic programming method to obtain explicit forms of the optimal consumption/portfolio and retirement time.

1. Introduction

In this paper, we consider the optimal consumption/portfolio and retirement choice problem of an economic agent with subsistence consumption constraints before retirement in a labor–disutility framework. An optimal consumption/portfolio problem concerning downside constraints on consumption rate process and terminal wealth was introduced by Lakner and Nygren [7] and the explicit solution was obtained through Malliavin calculus. With the help of the dynamic programming principle, Gong and Li [4] investigated the role of index bonds in optimal consumption/portfolio with a subsistence consumption constraint for a CRRA utility function of consumption. For the general utility function of consumption, Shin et al. [14] obtained explicit forms of the optimal consumption/portfolio with downside consumption constraints using the martingale method. Zariphopoulou [16], and Vila and Zariphopoulou [15] contributed the study on the optimal consumption/portfolio problem with borrowing constraints.

The optimal consumption/portfolio problem of an agent with voluntary retirement in a labor–disutility framework was pioneered by Choi and Shim [1]. Using the dynamic programming method and solving the free

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boundary value problem, Choi and Shim [1] provided explicit forms of the optimal consumption/portfolio and retirement time with the general utility function. Another analysis on an optimal consumption/portfolio and retirement choice problem was accomplished by Farhi and Panageas [3] in a labor–leisure framework. Farhi and Panageas [3] and Shin [13] assumed that the leisure rate process is binomial (l_1 prior to retirement and $\bar{l}$ after retirement) and the utility function of consumption and leisure is of Cobb–Douglas type. Choi et al. [2] studied a similar model but a general CES utility function, of which a Cobb–Douglas utility function is a special case, was adopted and leisure rate process before retirement was assumed to be a control variable.

An optimal consumption/portfolio and retirement choice problem with subsistence consumption constraints was firstly investigated by Lim et al. [9] in a labor–disutility framework using the martingale method. Lee and Shin [8] solved the problem in a labor–leisure framework using the dynamic programming method. In this paper, we solve the problem in a labor–disutility framework using the dynamic programming method developed by Karatzas et al. [5] and provide further analysis on the threshold wealth level corresponding to the optimal retirement time.

The remainder of this paper proceeds as follows. Section 2 introduces the financial market model and in Section 3, we describe the optimization problem of the agent. In Section 4, we solve the optimization problem using the dynamic programming method and provide an analysis on the threshold wealth level corresponding to the optimal retirement time.

2. The economy

We assume that the prices of the riskless asset P_t and the risky asset S_t evolve according to the following equations: $dP_t/P_t = rdt$, and $dS_t/S_t = \mu dt + \sigma dB_t$, respectively, where B_t is a standard Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\{\mathcal{F}_t\}_{t\geq 0}$ is the $\mathbb{P}$-augmentation of the filtration generated by the standard Brownian motion $\{B_t\}_{t\geq 0}$. r, μ and σ are assumed to be constant. We denote the amount invested in the risky asset at time t by π_t and the rate of consumption at time t by c_t. $\boldsymbol{\pi} := \{\pi_t\}_{t\geq 0}$ is a portfolio process if π_t is measurable and adapted to $\{\mathcal{F}_t\}_{t\geq 0}$ and $\mathbf{c} := \{c_t\}_{t\geq 0}$ is a consumption rate process if c_t is measurable, nonnegative and adapted to $\{\mathcal{F}_t\}_{t\geq 0}$. In addition, they satisfy the conditions

$$\int_0^t \pi_s^2 ds < \infty \quad \text{and} \quad \int_0^t c_s ds < \infty, \text{ for all } t \geq 0 \text{ a.s.}$$

The agent receives labor income until retirement at the constant labor income rate denoted by $\epsilon > 0$. Let τ be the retirement time from labor, which is an $\mathcal{F}_t$-stopping time. The agent is endowed with the initial amount of wealth $X_0 = x$. Thus, the agent's wealth process X_t at time t evolves according to the following equation

$$dX_t = \left[rX_t + \pi_t(\mu - r) - c_t + \epsilon \mathbf{1}_{\{0\leq t<\tau\}}\right] dt + \pi_t \sigma dB_t. \tag{2.1}$$

3. The optimization problem

We consider a situation where the agent is subject to subsistence consumption constraints only before retirement. We denote a positive subsistence level of consumption rate before retirement by $R > 0$. The agent's objective is to maximize the total expected discounted utility from consumption with the following constraint:

$$c_t \geq R, \text{ for all } 0 \leq t < \tau. \tag{3.1}$$