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A Filippov system describing the effect of prey refuge use on a ratio-dependent predator–prey model

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ABSTRACT

In this paper, a Filippov ratio-dependent predator–prey model is proposed to describe the effect on behavioral refuges caused by prey instinct anti-predator behavior. The proposed model extends the classical ratio-dependent predator–prey model by combining a prey–predator ratio that describes the behavioral refuges make sense once it is less than a certain threshold. One of the prominent mathematical features of our model, distinguishing from the classical one, is that there exist singular points on discontinuous surface whose characteristics determine the main dynamical behaviors of this model. The complete analysis of topological structures of orbits near all the singular points is presented. We show 14 types of system behaviors realized for various parameter values. In particular, globally stable pseudo-equilibrium and globally finite time stable canard cycle are shown to exist in some ranges of parameter values, which cannot be achieved in classical model.

1. Introduction

The hiding behavior of prey in dynamics of predator–prey interaction has stabilizing effect on the interaction and prey extinction can be prevented by refuges use. One of such experimental evidences was given by Gause et al. in 1936 who observed in an experiment with paramecia and yeast cells how interaction between the two species depends on the concentration of yeast cells [11]. There exists a certain threshold concentration of yeast cells sediment on the bottom which is not accessible to paramecia. When the concentration of yeast cells exceeds the threshold, the exceeding yeast cells will appear on the surface of medium accessible to paramecia. In this experiment, periodic fluctuations were observed in the concentrations of both species. To explain the observation mathematically, Gause et al. replaced the linear consumption rate used in Lotka–Volterra predator–prey model by a discontinuous saturating function which is zero below the

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threshold prey concentration and jumps at this threshold. Hereafter, we say this model as Guase model. The numerical simulation of Gause model shows the character of periodic fluctuation. Recently, this model was re-analyzed by Krivan [14] and Tang and Liang [19] using Filippov theory which was not available at the time when Gause and his co-workers analyzed their model. It is likely that Gause model is the first predator–prey model with discontinuous right hand side to describe the effect of prey refuge use.

To our best knowledge, no previous study has examined dynamics of prey refuge use for ratio-dependent predator–prey species by the model with discontinuous functional response. To fill this gap, we propose a Filippov ratio-dependent predator–prey model under the following assumptions: (i) whether preys move in or out refuge depends on the prey–predator ratio, i.e., there exists a prey–predator threshold ratio above which preys will move outside the refuge, otherwise they will stay in the refuge; (ii) resources for preys are mainly out of refuge, such that all the preys are out of refuge if prey–predator ratio is large; (iii) within refuge, resources for preys are scarce, but sufficient to keep preys alive and growing logistically; (iv) the prey is the only resource of predator. Under these assumptions, the (Filippov) ratio-dependent predator–prey model describing the effect of prey refuge use can be written as follows:

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}s} &= au(1-\frac{u}{k}) - c\phi(u,v),\\ \frac{\mathrm{d}v}{\mathrm{d}s} &= -dv + f\phi(u,v),\end{aligned} \tag{1.1}$$

where

$$\phi(u,v) = \begin{cases} 0, & \frac{u}{v} < \lambda,\\ \frac{uv}{mv+u}, & \frac{u}{v} > \lambda,\end{cases}$$

u and v are the number of preys and predators, respectively; λ is the prey–predator threshold ratio and a, k, c, m, f and d are positive constants which represent prey intrinsic growth rate, carrying capacity, capturing rate, half capturing saturation constant, conversion rate and predator death rate, respectively.

When the prey–predator ratio is below the threshold ratio ($\frac{u}{v} < \lambda$), preys stay in the refuge and grow logistically, while predators die exponentially. That is,

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}s} &= au(1-\frac{u}{k}),\\ \frac{\mathrm{d}v}{\mathrm{d}s} &= -dv.\end{aligned} \tag{1.2}$$

When the prey–predator ratio is above the threshold ratio ($\frac{u}{v} > \lambda$), population dynamics are governed by the ratio-dependent prey–predator model

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}s} &= au(1-\frac{u}{k}) - \frac{cvu}{mv+u},\\ \frac{\mathrm{d}v}{\mathrm{d}s} &= v[\frac{fu}{mv+u} - d].\end{aligned} \tag{1.3}$$

The classical ratio-dependent predator–prey model (1.3) has been extensively studied since Arditi and Ginzburg [2–4,15,20]. Since such a model can avoid the biological control paradox reported in Luck [16], it has been accepted more and more widely. In [1], the effect of refuge use was incorporated into model (1.3) with continuous right hand side. The author of [1] predicted the periodic fluctuation, but they overestimated the amplitudes of both prey and predator. One possible reason for this might be due to ignorance of the threshold in refuge use. Thus, we hope our model (1.1) incorporated with the threshold phenomenon