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Optimal contracting with moral hazard and behavioral preferences ☆

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ABSTRACT

We consider a continuous-time principal–agent model in which the agent's effort cannot be contracted upon, and both the principal and the agent may have non-standard, cumulative prospect theory type preferences. We find that the optimal contracts are likely to be "more nonlinear" than in the standard case with concave utility preferences. In the special case when the principal is risk-neutral, we show that she will offer a contract which effectively makes the agent less risk averse in the gain domain and less risk seeking in the loss domain, in order to align the agent's risk preference better with the principal's.

1. Introduction

In this paper we consider optimal contracting between two parties – the principal ("she") and the agent ("he") – in continuous time, when the effort of the agent cannot be contracted upon. Cvitanić, Wan and Zhang [3] develop a theory for general concave utility functions for the two parties. Motivated by behavioral criteria, specifically that of the (cumulative) prospect theory (CPT; Kahneman and Tversky [9], Tversky and Kahneman [18]), in the present paper we go a step further and allow the principal and the agent to have non-concave, CPT type preference functions.

Our model studies the case of "hidden actions" or "moral hazard", in which the agent's control (effort) of the drift of the output process cannot be contracted upon, either because it is unobserved by the principal,

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and/or because it is not legally enforceable. Hence, the contract is a function of only terminal values of the underlying output process.

The seminal paper on the continuous-time principal–agent problems is [8]. In that paper the principal and the agent have exponential utility functions and the optimal contract is linear. Their work was generalized and extended by Schättler and Sung [14,15], Sung [16,17], Müller [10,11], and Hellwig and Schmidt [7]. The papers by Williams [19] and Cvitanić, Wan and Zhang [3] use the stochastic maximum principle and forward–backward stochastic differential equations (FBSDEs) to characterize the optimal compensation for more general utility functions, under moral hazard. Cvitanić and Zhang [4] and Carlier, Ekeland and Touzi [2] consider also the adverse selection case of "hidden type", in which the principal does not observe the "intrinsic type" of the agent. Sannikov [13] re-awakens the interest in the continuous-time principal–agent problem by finding a tractable model for solving the problem with a random time of retiring the agent and with continuous payments to the agent.

Optimal contracting with cumulative prospect theory (CPT) preferences, and, in particular, with the agent being loss-averse, has already been studied in [6]. They calibrate the model to CEO compensation data and find that it explains better the observed compensation contracts than the standard utility preferences (risk-aversion) model. This shows the usefulness of studying such models.

There are two main contributions of our paper. First, we show that the optimal payoff depends in a nonlinear way on the value of the output at the time of payment, and may be "more nonlinear" than with the standard, concave preferences. Second, we prove that, with a risk-neutral principal, and under some technical conditions, the optimal contract convexifies the agent's preference function if it is a classical concave utility function; and the optimal contract convexifies the agent's preference function in the gain part and concavifies it in the loss part if the agent has an S-shaped behavioral utility function.

We then study in details examples with a risk-neutral principal, and an agent who has piecewise logarithmic or power objective functions respectively. Notably, we find that the CPT preferences "increase" the nonlinearity of the optimal contract, thus providing an additional rationale for the existence of option-like contracts in practice.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents the general approach to solving the agent's and the principal's optimization problems. Section 4 studies the case of a risk-neutral principal. Section 5 provides detailed examples. Finally, we conclude with Section 6.

2. The model

We would like to have a model for the output process X of the form

$$dX_t = u_t v_t dt + v_t dW_t \tag{1}$$

for a Brownian motion process W, where u_t represents the effort of the agent, and v_t is the volatility process. As is usually noted in contract theory, choosing u is equivalent to choosing a probability measure over the underlying probability space. Thus, we proceed by developing the following weak formulation of the model (see [5] for more on the weak formulation).

Let B be a standard Brownian motion under some probability space with probability measure Q, and $\mathbf{F}^B = \{\mathcal{F}_t^B\}_{0\le t\le T}$ be the filtration on $[0,T]$ generated by B and augmented by Q-null sets. For any $\mathbf{F}^B$-adapted square integrable process v, let

$$X_t \triangleq x_0 + \int_0^t v_s dB_s \tag{2}$$