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The comparison of the Riemann solutions in gas dynamics ☆

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ABSTRACT

In this paper, we compare two Riemann solution constructions of isentropic and non-isentropic gas dynamic equations, with the same initial values. We obtain that when rarefaction waves occur in the non-isentropic solution, the same family rarefaction waves occur in the isentropic solution, however, when advancing or reflected shock waves occur in the isentropic case, the same kind shock waves occur in the non-isentropic case. Moreover, by numerical calculation, it can be found that the wave intensities of the two solutions are quite close. Our conclusion reflects that almost non-isentropic gas dynamic process can be simulated by isentropic gas dynamic process in practice.

1. Introduction

We consider the equations of isentropic gas dynamics and non-isentropic gas dynamics with the same Riemann initial data,

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p)_x = 0, \\ p(\rho) = k_0 \rho^\gamma, \gamma > 1, \\ k_0 = e^{s_0/c_v}, \end{cases} \tag{1.1}$$

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p)_x = 0, \\ [\rho(\frac{1}{2}u^2 + \mathbf{e})]_t + [\rho u(\frac{1}{2}u^2 + \mathbf{e}) + pu]_x = 0, \\ p(s, \rho) = e^{s/c_v} \rho^\gamma, \gamma > 1, \end{cases} \tag{1.2}$$

where ρ, u and p are density, velocity and pressure respectively, $\mathbf{e}$ and s are internal energy and entropy. k_0, c_v and γ are positive constants, the initial data are all

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$$U_0 = \begin{cases} (\rho_l, s_l, u_l) & x < 0, \\ (\rho_r, s_r, u_r) & x > 0 \end{cases} \tag{1.3}$$

and $s_l = s_r = s_0$.

We prove that when advancing or reflected shock waves occur in isentropic case, the same kind shock waves occur in non-isentropic case, when rarefaction waves occur in non-isentropic case, the homologous rarefaction waves occur in isentropic case. Furthermore, we take the numerical simulation for the wave intensities. The result shows that some non-isentropic gas dynamic process can be simulated by isentropic gas dynamic process.

Before we detail more precisely our results, we recall the research of one-dimensional gas dynamic equations in Eulerian coordinate, which contain the isentropic case and the non-isentropic case.

The framework of the existence of the solution for isentropic gas dynamic equations with Cauchy data is almost completed by using the compensated compactness theory [14] and the Lax–Friedrichs scheme; DiPerna [6,7] established the existence of the weak entropy solution for the isentropic Euler equations with general L^∞ initial data for $\gamma = 1 + \frac{2}{2n+1}$, $n \geq 2$ and n is integer; Ding, Chen and Luo [11,4] also got the existence of the isentropic solution by vanishing numerical viscosity for $\gamma \in [1, \frac{5}{3}]$; Lions, Perthame, Tadmor and Souganidis [1,10,9] got the existence results for $\gamma > 3$; Huang and Wang [8] got the existence results for $\gamma = 1$.

On the contrary, the framework of the existence of the solution for non-isentropic gas dynamic equations with Cauchy data is far from being constructed. The compensated compactness theory and the Lax–Friedrichs scheme encountered bottlenecks. Researchers tried to find the connection and the difference between isentropic case and non-isentropic case, by using the stability results of Bressan and Colombo [2] for strictly hyperbolic 2×2 systems in one space dimension, L. Saint-Raymond [12] proved that the solutions of isentropic and non-isentropic Euler equations in one space dimension with the respective initial data (ρ_0, u_0) and (ρ_0, u_0, θ_0), where $\theta_0 = \rho_0^{\gamma-1}$ remain close as soon as the total variation of (ρ_0, u_0) is sufficiently small. Chen, S.X. and Geng, J.B. promoted L. Saint-Raymond's conclusion [12]. The similar results also can be found in [15], Chen, G.Q. [5] proved that the adiabatic exponent $\gamma \to 1$ that passes from the non-isentropic to isothermal Euler equations.

Actually, when shock waves occur, the entropy increases after the shock passes. This contradiction between the isentropic and non-isentropic gas dynamic processes is just our research interests. By comparing the similarities and differences between the isentropic and non-isentropic gas dynamic processes, we obtain that though the isentropic gas dynamic process is more like an approximate model, almost non-isentropic gas dynamic process can be simulated by isentropic gas dynamic process in practice. In other words, the results of isentropic gas dynamic process are also important in practical applications.

Furthermore, we can also obtain similar conclusion by investigating the non-isentropic equations with initial data that $s_l \neq s_r$, which drives us to consider the simulation of the large time asymptotic behavior for non-isentropic gas dynamics with general Cauchy data by isentropic case in the future. These study may provide some enlightenments in the research of non-isentropic gas dynamic equation with Cauchy data.

Our paper is divided into three parts. The first one is about solving the Riemann problems of isentropic and non-isentropic gas dynamic equations, with equal initial values. By using the similar idea in [13] and operating the two problems in (ρ, u) plane, we rewrite the solutions of these two Riemann problems in different form. The second one is about obtaining the dividing curves of the solutions of two problems respectively—R_1, S_1, R_2, S_2 for isentropic case and $\tilde{R}_1$, $\tilde{R}_3$, $\tilde{R}_1 + \tilde{J}_2$, $\tilde{J}_2 + \tilde{R}_3$ for non-isentropic case, determine their spatial relationships with each other and study the connection of the two problems. The third one is about the numerical calculation and image display. We take the construction of the comparison about two problems with different γ and calculate the gap about two problems.