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Existence of a mixed equilibrium for a compact generalized strategic game

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ABSTRACT

In this paper, we first introduce new concepts of generalized mixed strategic game and mixed equilibrium which generalize the mixed equilibrium for a compact strategic game due to Nash. Next, by using an equilibrium existence theorem for the compact convex strategic game $\mathcal{G} = (X_i; T_i, f_i)$, we prove a mixed equilibrium existence theorem for the generalized mixed strategic game $\hat{\mathcal{G}} = (\Delta(X_i); \hat{T}_i, \hat{f}_i)$ without assuming the convexity of strategy space X_i. Finally, two examples of the Nash equilibrium and mixed equilibrium for generalized strategic games $\mathcal{G} = (X_i; T_i, f_i)$ are given.

1. Introduction

In 1950, Nash [19] first established a pioneering equilibrium existence theorem by using the Kakutani fixed point theorem. Next, by applying the Eilenberg–Montgomery fixed point theorem, Debreu [10] established a generalization of the Nash equilibrium existence theorem which assumes the best utility functions and response profile correspondences. Since then, the classical results of Nash [19] and Debreu [10] have served as basic references for the existence of Nash equilibrium for non-cooperative generalized games. In all of them, compactness and convexity of strategy spaces, continuity and convexity of the payoff functions, and continuity of the constraint correspondences were assumed.

Till now, there have been a number of generalizations and applications of the Nash equilibrium existence theorem and Debreu equilibrium existence theorem in several areas by relaxing the assumptions on compactness and convexity of strategy spaces, continuity and convexity of the payoff functions, and also continuity of the constraint correspondences, e.g., see Aliprantis [1], Ansari et al. [4], Ansari and Lin [5], Arrow and Debreu [6], Aubin [7], Friedman [11], Lin et al. [16], Lin and Ansari [15], Osborne and Rubinstein [20], Shafer and Sonnenschein [21], and references therein. Among those generalizations, using their fixed point theorems and coincidence theorems, Ansari et al. [4] and Lin et al. [16] proved general equilibrium existence

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theorems for generalized abstract economies by relaxing compactness assumptions on the strategy spaces and assuming the mild conditions on the best utility functions. Also, Im and Kim [14] proved the equilibrium existence theorem for the noncompact generalized game by applying the Himmelberg fixed point theorem without assuming the assumptions on best utility functions and response profile M_i in [10].

The notion of a non-cooperative Nash equilibrium is central to game theory. When a pure strategy equilibrium (in which each player chooses with certainty one of his strategies) does not exist, attention typically turns to a mixed strategy equilibrium where each player chooses a probability distribution over his strategies. Since the set of mixed strategies includes that of pure strategies as a subset, it is perhaps not surprising that a mixed-strategy equilibrium may exist even if a pure-strategy equilibrium does not exist. For a discussion of mixed strategies and the corresponding equilibrium, see Friedman [11], Moulin [17], and von Neumann and Morgenstern [23]. In fact, the most commonly used argument for considering mixed strategies was first provided by von Neumann and Morgenstern, who argued that "playing several different strategies at random" was effective way of protecting player against "having his intension found out by his opponent". From this point of view, the adoption of mixed strategies (as opposed to pure strategies) raises the adoption of players' security levels as measured by expected utility. There might be some doubt about the effectiveness of the mixed strategies and the resultant equilibria as in Shubik [22] and Moulin [17]; however, there have been numerous positive results on the effectiveness of the mixed strategies and the resultant equilibria as shown by Aliprantis et al. [3] and some game theorists.

In this paper, we first introduce new concepts of generalized mixed strategic game and mixed equilibrium which generalize the mixed equilibrium for a compact strategic game due to Nash [19]. Next, by using the equilibrium existence theorem for a compact convex game $\mathcal{G} = (X_i; T_i, f_i)$ in [14], we prove the existence theorem of mixed equilibrium for a compact non-convex generalized mixed strategic game $\hat{\mathcal{G}} = (\Delta(X_i); \hat{T}_i, \hat{f}_i)$ which generalizes the existence theorem of mixed equilibrium for the mixed strategic game $\hat{\mathcal{G}} = (\Delta(X_i); \hat{f}_i)$ due to Nash [19] and Aliprantis et al. [3]. Finally, two examples of the existence of Nash equilibrium and mixed equilibrium for generalized strategic games $\mathcal{G} = (X_i; T_i, f_i)$ are given.

2. Preliminaries

We begin with some notations and definitions. If A is a nonempty set, we shall denote by 2^A the family of all subsets of A. If A is a subset of a vector space, we shall denote by $co\, A$ the convex hull of A. Let E be a topological vector space and A, X be nonempty subsets of E. If $T : A \to 2^E$ and $S : A \to 2^X$ are multimaps (or correspondences), then $co\, T : A \to 2^E$ and $S \cap T : A \to 2^X$ are correspondences defined by $(co\, T)(x) = co\, T(x)$, $(S \cap T)(x) = S(x) \cap T(x)$ for each $x \in A$, respectively.

Let $I = \{1, 2, \dots, n\}$ be a finite (or possibly countably infinite) set of players, and let X_i be a nonempty topological space as an action space for each $i \in I$, and denote $X_{-i} := \prod_{j \in I \setminus \{i\}} X_j$. For an action profile $x = (x_1, \cdots, x_n) \in X = \prod_{i \in I} X_i$, we shall write $x_{-i} := (x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n) \in X_{-i}$, and we simply write $x := (x_{-i}, x_i) = (x_1, \cdots, x_{i-1}, x_i, x_{i+1}, \cdots, x_n) \in X$.

Now we recall basic definitions of continuities concerned with multimaps. Let X, Y be nonempty topological spaces and $T : X \to 2^Y$ be a multimap. A multimap $T : X \to 2^Y$ is said to be *lower semicontinuous* if for each $x \in X$ and each open set V in Y with $T(x) \cap V \neq \emptyset$, there exists an open neighborhood U of x in X such that $T(y) \cap V \neq \emptyset$ for each $y \in U$; and a multimap $T : X \to 2^Y$ is said to be *upper semicontinuous* if for each $x \in X$ and each open set V in Y with $T(x) \subset V$, there exists an open neighborhood U of x in X such that $T(y) \subset V$ for each $y \in U$. A multimap T is said to be *continuous* if T is both lower semicontinuous and upper semicontinuous. It is also known that $T : X \to 2^Y$ is lower semicontinuous if and only if for each closed set V in Y, the set $\{x \in X \mid T(x) \subset V\}$ is closed in X. If a multimap $T : X \to 2^Y$ is upper semicontinuous with closed values, then T has a closed graph. The converse is true whenever Y is compact. For the sequential definitions of the upper and lower semicontinuities, see Aubin [7] or Aliprantis and Border [2].