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Investigation of the analyticity of dissipative–dispersive systems via a semigroup method

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ABSTRACT

In this work, we study the analyticity of Kuramoto–Sivashinsky type equations and related systems by exploring the applicability of the semigroup method, which was developed in Collet et al. [5]. We establish the analyticity, with respect to the spatial variable in a strip around the real axis, for a variety of dissipative–dispersive systems, which possess universal attractors. We also provide lower bounds for the width of the strip of analyticity.

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1. Dissipative–dispersive systems

A simple such system is the dispersively modified Kuramoto–Sivashinsky (KS) equation

$$u_t + uu_x + u_{xx} + \nu u_{xxxx} + \mathcal{D}u = 0, \tag{1.1}$$

defined on 2π-periodic intervals, where ν a positive constant and $\mathcal{D}$ a linear antisymmetric pseudo-differential operator; in Fourier space

$$(\widehat{\mathcal{D}w})_k = \mathrm{i}d_k\hat{w}_k, \quad d_{-k} = -d_k \in \mathbb{R}, \tag{1.2}$$

that is, $\mathcal{D}$ is dispersive. An equation of the form of (1.1) has been derived in the context of interfacial hydrodynamics. For example, in Papageorgiou et al. [20] and Kas-Danouche et al. [13] an equation of this type describes the dynamics of core-annular film flows with applications to oil transport (lubricated pipe-lining). In the latter derivation $\widehat{\mathcal{D}}$ is expressed as

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$$\widehat{\mathcal{D}u}(\xi) = \frac{\mathrm{i}\xi^2 I_1(\xi)}{\xi I_1^2(\xi) - \xi I_0^2(\xi) + 2I_0(\xi)I_1(\xi)}\hat{u}(\xi),$$

where $I_n(\xi)$ denotes the modified Bessel function of the first kind of order n.

For (one-dimensional) falling film flows a particular case of (1.1) where $\mathcal{D}u = \delta u_{xxx}$ and δ is a constant was originally derived by Topper and Kawahara [24]. The resulting equation is the following KS/KdV equation (also known as Kawahara equation)

$$u_t + uu_x + u_{xx} + \delta u_{xxx} + \nu u_{xxxx} = 0, \tag{1.3}$$

and note that the dispersive term is of lower order than the stabilizing term u_{xxxx}. Kawahara and Toh [14] were among the first to establish numerically the regularizing effect of dispersion on the dynamics with traveling wave pulses emerging at large times.

It is noteworthy that Eq. (1.3) with the inclusion of a fifth order dispersion term

$$u_t + uu_x + u_{xx} + \delta u_{xxx} + \nu u_{xxxx} + \varepsilon u_{xxxxx} = 0, \tag{1.4}$$

known as the Benney–Lin equation, has been derived in the context of the one-dimensional evolution of sufficiently small amplitude long waves in various problems in fluid dynamics. (See, for example, Benney [2] and Lin [17].)

The global well-posedness of the periodic initial value problem for (1.1) can be derived from the work of Tadmor [23]. In particular, it can be also shown that this initial value problem possesses a global (space periodic) solution which grows at most exponentially in time. Existence of a universal attractor, bounded in every Sobolev norm, was established by Frankel and Roytburd in [7,8]. Global well-posedness of the periodic initial value problem for (1.4) with initial data in $H^s_{\mathrm{per}}(\mathbb{R})$, $s \geq 0$, has been established by Biagioni and Linares [3].

It is noteworthy that in the case of vanishing dispersion, i.e., $\mathcal{D} \equiv 0$, Eq. (1.1) reduces to the well-known KS equation

$$u_t + uu_x + u_{xx} + u_{xxxx} = 0, \tag{1.5}$$

defined on L-periodic intervals, which is one of the simplest nonlinear PDEs exhibiting complex spatio-temporal dynamics, and which has been derived in the context of plasma ion mode instabilities by LaQuey et al. [16], reaction-diffusion systems by Kuramoto and Tsuzuki [15], laminar flame fronts by Sivashinsky [21] and viscous liquid flows on an inclined plane by Sivashinsky and Michelson [22].

Note that equation

$$u_t + uu_x + u_{xx} + \nu u_{xxxx} = 0, \tag{1.6}$$

defined on 2π-periodic intervals, is obtained, from Eq. (1.5) given on L-periodic intervals by the rescaling (dropping the bars):

$$\bar{t} = \nu t, \qquad \bar{x} = \nu^{1/2}x, \qquad \bar{u} = \nu^{-1/2}u, \tag{1.7}$$

where $\nu = (2\pi/L)^2$. Let us explain how this rescaling works. First, by setting $\bar{x} = (2\pi/L)x$ we see that the interval $[0, L]$ transformed to $[0, 2\pi]$. Also by using

$$\frac{\partial}{\partial x} \longmapsto \frac{2\pi}{L}\frac{\partial}{\partial \bar{x}},$$

in (1.5), we get

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