J. Math. Anal. Appl. 420 (2014) 1301–1315

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Global well-posedness for the generalized magneto-hydrodynamic equations in the critical Fourier–Herz spaces ☆

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ARTICLE INFO

Article history:
Received 22 October 2013
Available online 19 June 2014
Submitted by P.G. Lemarie-Rieusset

Keywords:
Generalized magneto-hydrodynamic equations
Global well-posedness
Ill-posedness
Fourier–Herz spaces

ABSTRACT

This paper concerns the Cauchy problem of the n-dimensional generalized incompressible magneto-hydrodynamic (GMHD) equations with $\beta \in (\frac{1}{2}, 1]$. By using the Fourier localization argument and the Littlewood–Paley theory, we get the global well-posedness of the GMHD equations with small initial data (u_0, b_0) belongs to the critical Fourier–Herz spaces $\dot{\mathcal{B}}_q^{-(2\beta-1)}$ with $q \in [1, 2]$. In addition, for $2 < q \leq \infty$, ill-posedness for the case $\beta = 1$ in $\dot{\mathcal{B}}_q^{-1}$ is also established.

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1. Introduction

In this paper, we consider the following Cauchy problem of the n-dimensional generalized incompressible magneto-hydrodynamic (GMHD) equations in $\mathbb{R}^n \times (0, +\infty)$, $n \geq 3$:

$$u_t + (-\Delta)^\beta u + (u \cdot \nabla)u - (b \cdot \nabla)b + \nabla P = 0, \quad (x, t) \in \mathbb{R}^n \times (0, +\infty), \tag{1.1}$$

$$b_t + (-\Delta)^\beta b + (u \cdot \nabla)b - (b \cdot \nabla)u = 0, \quad (x, t) \in \mathbb{R}^n \times (0, +\infty), \tag{1.2}$$

$$\operatorname{div} u = 0, \qquad \operatorname{div} b = 0, \quad (x, t) \in \mathbb{R}^n \times (0, +\infty) \tag{1.3}$$

with the initial conditions

$$u(x, 0) = u_0(x), \qquad b(x, 0) = b_0(x), \quad x \in \mathbb{R}^n, \tag{1.4}$$

☆ Q. Liu is partially supported by the Hunan Provincial Natural Science Foundation of China (13JJ4043) and NNSF of China (11326155, 11171357); J. Zhao is partially supported by NNSF of China (11171357, 11371294) and the Fundamental Research Funds for the Central Universities (2014YB031).

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http://dx.doi.org/10.1016/j.jmaa.2014.06.031
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where $u = u(x,t) = (u^1(x,t),\dots,u^n(x,t))$, $b = b(x,t) = (b^1(x,t),\dots,b^n(x,t))$ and $P = P(x,t) = p(x,t) + \frac{|b|^2}{2}$ stand for the fluid velocity field, the magnetic field, and the total kinetic pressure, respectively, and $\beta \in (\frac{1}{2},1]$. The fractional Laplacian operator $(-\Delta)^\beta$ with respect to space variable x is a Riesz potential operator defined as usual through Fourier transform as $\mathcal{F}((-\Delta)^\beta f)(\xi) = |\xi|^{2\beta}\mathcal{F}f(\xi)$, where $\mathcal{F}f(\xi) = \widehat{f}(\xi) = \frac{1}{\sqrt{2\pi}^n}\int_{\mathbb{R}^n} e^{-ix\cdot\xi}f(x)\mathrm{d}x$. The initial velocity field u_0 and the initial magnetic field b_0 satisfying $\operatorname{div} u_0 = 0$, $\operatorname{div} b_0 = 0$, respectively. For simplicity, we set the Reynolds number, the magnetic Reynolds number, and the corresponding coefficients to be equal to one.

The GMHD system (1.1)–(1.4), which describes the macroscopic behavior of the electrically conducting incompressible fluids in a magnetic field, is a generalization of the usual incompressible MHD system by replacing the Laplacian operator $-\Delta$ in the MHD system with a general fractional Laplacian operator $(-\Delta)^\beta$ (see Wu [22,23,25]). When $\beta = 1$, the system (1.1)–(1.4) becomes the usual MHD system, which has drawn much attention during the past twenty more years (cf. [4,6,11,19,20] and the references cited therein). In particular, when $\beta = 1$ and $b \equiv 0$, the system (1.1)–(1.4) becomes the well-known Navier–Stokes equations, for which there have been a lot of works concerning well-posedness of the initial value problem in various classical function spaces. For instance, Fujita and Kato [9] proved both the global well-posedness for small initial data and the local well-posedness for large initial data in the Sobolev space $H^s(\mathbb{R}^n)$ with $s \geq n/2-1$. Kato [13] established similar results in the Lebesgue spaces $L^n(\mathbb{R}^n)$. Giga and Miyakawa [10] considered the Cauchy problem in $L^p(\Omega)$, where Ω is a bounded domain and $p \geq n$. Cannone [3] obtained the global solutions in the case of $n = 3$ for initial data $u_0 \in B^{-1+3/q}_{q,\infty}(\mathbb{R}^3)$ $(3 < q \leq 6)$. Koch and Tataru [14] studied local solutions for initial data $u_0 \in vmo^{-1}$ and global solutions for small initial data $u_0 \in BMO^{-1}$. Very recently, Cannone and Wu [5] studied the global well-posedness in the critical Fourier–Herz space $\dot{\mathcal{B}}^{-1}_q$ with $1 \leq q \leq 2$. In papers [2,27], the ill-posedness of the Navier–Stokes equations is observed.

For the GMHD system (1.1)–(1.4), Wu in [22] established the global-in-time weak solution for any given divergence free initial value $(u_0,b_0) \in L^2(\mathbb{R}^n)$. Moreover, when $\beta \geq \frac{1}{2}+\frac{n}{4}$, Wu proved that the weak solution is in fact the classical solution. Yuan [28] obtained the local-in-time existence and uniqueness of smooth solution for any sufficient smooth initial data (u_0,b_0). Liu, Zhao and Cui [17] obtained the global existence and stability of solutions for system (1.1)–(1.4) with small initial data (u_0,b_0) belonging to the pseudomeasure space PM^a (with $a \geq n-(2\beta-1)$ a given parameter), where PM^a is defined by

$$PM^a := \Big\{ f \in \mathcal{S}' : \widehat{f} \in L^1_{loc}(\mathbb{R}^n),\ \|f\|_{PM^a} := \operatorname*{ess\,sup}_{\xi\in\mathbb{R}^n} |\xi|^a \big|\widehat{f}(\xi)\big| < \infty \Big\}.$$

We also refer to [18,23–25,29,30] on other related topics on system (1.1)–(1.4).

Recall that system (1.1)–(1.4) is invariant under a particular change of space and time scaling. More precisely, if (u,b,P) solves system (1.1)–(1.3) with initial data (u_0,b_0), then the re-scaled functions $(u_\lambda,b_\lambda,P_\lambda)$ for all $\lambda > 0$, where

$$u_\lambda(x,t) = \lambda^{2\beta-1}u\big(\lambda x,\lambda^{2\beta}t\big), \qquad b_\lambda(x,t) = \lambda^{2\beta-1}b\big(\lambda x,\lambda^{2\beta}t\big), \qquad P_\lambda(x,t) = \lambda^{4\beta-2}P\big(\lambda x,\lambda^{2\beta}t\big),$$

also solves the system (1.1)–(1.3) with initial data $(u_{0\lambda},b_{0\lambda}) := (\lambda^{2\beta-1}u_0(\lambda x),\lambda^{2\beta-1}b_0(\lambda x))$. This scaling invariance property is particularly significant for (1.1)–(1.4) and naturally leads to the definition of critical space for system (1.1)–(1.4). A function space is critical for (1.1)–(1.4) if it is invariant under the scaling

$$f_\lambda(x) := \lambda^{2\beta-1}f(\lambda x). \tag{1.5}$$

It is easy to verify that the spaces $\dot{H}^{\frac{n}{2}-(2\beta-1)}(\mathbb{R}^n)$, $L^{\frac{n}{2\beta-1}}(\mathbb{R}^n)$ and $\dot{B}^{-(2\beta-1)+\frac{n}{p}}_{p,q}(\mathbb{R}^n)$ $(n < p \leq \infty,\ 1 \leq q \leq \infty)$ are critical spaces for (1.1)–(1.4). We notice that the Fourier–Herz space $\dot{\mathcal{B}}^{-(2\beta-1)}_q$, which will be discussed in this paper, is also a critical space (see Remark 2.2 below).

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