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Subspaces of codimension two with large projection constants

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This paper is dedicated to Eusebio Corbacho Rosas, advisor of the first author, on the occasion of his 65th birthday

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ABSTRACT

Let V be an n-dimensional real Banach space and let $\lambda(V)$ denote its absolute projection constant. For any $N \in \mathbb{N}$, $N \geq n$ define

$$\lambda_n^N = \sup\{\lambda(V) \colon \dim(V) = n,\ V \subset l_\infty^{(N)}\}.$$

The aim of this paper is to determine minimal projections with respect to l_1-norm as well as with respect to l_∞-norm for subspaces given by solutions of certain extremal problems. As an application we show that for any $n, N \in \mathbb{N}$, $N \geq n$ there exists an n-dimensional subspace $V_n \subset l_1^{(N)}$ such that

$$\lambda_n^N = \lambda(V_n, l_1^{(N)}).$$

Also we calculate relative and absolute projection constants of some subspaces of codimension two in $l_1^{(N)}$ and $l_\infty^{(N)}$ for $N \geq 3$ being odd natural number. Moreover, we show that for any odd natural number $n \geq 3$,

$$\lambda_n^{n+1} < \max_{x \in [0,1]} f_n(x) \leq \lambda_n^{n+2},$$

where

$$f_n(x) = \frac{2n}{n+1}(1-x) + \frac{1}{2}\left(x - 2\frac{1-x}{n+1} + \sqrt{\left(2\frac{1-x}{n+1} - x\right)^2 + 4(1-x)x}\right).$$

Also for any $n \in \mathbb{N}$ $x_n \in [0,1]$ satisfying

$$f_n(x_n) = \max_{x \in [0,1]} f_n(x)$$

will be calculated.

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1. Introduction

Let X be a real Banach space and let $V \subset X$ be a finite-dimensional subspace. A linear, continuous mapping $P : X \to V$ is called *a projection* if $P|_V = id|_V$. Denote by $\mathcal{P}(X, V)$ the set of all projections from X onto V. Set

$$\lambda(V, X) = \inf\{\|P\|\colon\ P \in \mathcal{P}(X, V)\}$$

and

$$\lambda(V) = \sup\{\lambda(V, X)\colon\ V \subset X\}.$$

A projection $P_o \in \mathcal{P}(X, V)$ is called *minimal* if

$$\|P_o\| = \lambda(V, X).$$

The constant $\lambda(V, X)$ is called the *relative projection constant* and $\lambda(V)$ the *absolute projection constant.* Minimal projections in the context of functional analysis and approximation theory have been extensively studied by many authors (see e.g., [1–16,18–31,33–35]). Mainly the problems of existence of minimal projections, uniqueness of minimal projections, finding concrete formulas for minimal projections and estimates of the constant $\lambda(V, X)$ were considered.

General bounds for absolute projection constants were studied by many authors (see e.g. [4–7,18–21,32]). It is well-known (see e.g. [36]) that if V is a finite-dimensional space then

$$\lambda(V) = \lambda(I(V), l_\infty),$$

where $I(V)$ denotes any isometric copy of V in l_∞. Denote for any $n \in \mathbb{N}$

$$\lambda_n = \sup\{\lambda(V)\colon\ \dim(V) = n\}$$

and for any $N \in \mathbb{N}$, $N \geq n$

$$\lambda_n^N = \sup\{\lambda(V)\colon\ V \subset l_\infty^{(N)}\}.$$

By the Kadec–Snobar Theorem (see [17]) $\lambda(V) \leq \sqrt{n}$ for any $n \in \mathbb{N}$. However, determination of the constant λ_n seems to be difficult.

The aim of this paper is to determine minimal projections with respect to l_1-norm as well as for l_∞-norm for subspaces given by solutions of certain extremal problems. As an application we show that for any $n, N \in \mathbb{N}$, $N \geq n$ there exists an n-dimensional subspace $V_n \subset l_1^{(N)}$ such that

$$\lambda_n^N = \lambda(V_n, l_1^{(N)}).$$

Also we show that for any odd natural number $n \geq 3$,

$$\lambda_n^{n+1} < \max_{x\in[0,1]} f_n(x) \leq \lambda_n^{n+2},$$

where

$$f_n(x) = \frac{2n}{n+1}(1-x) + \frac{1}{2}\left(x - 2\frac{1-x}{n+1} + \sqrt{\left(2\frac{1-x}{n+1} - x\right)^2 + 4(1-x)x}\right).$$