J. Math. Anal. Appl. 420 (2014) 1455–1467

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Stabilization of a joint-leg-beam system with boundary damping

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ARTICLE INFO

Article history:
Received 22 October 2013
Available online 19 June 2014
Submitted by D.L. Russell

Keywords:
Joint-leg-beam
Semigroup
Stabilization

ABSTRACT

In this paper we study a model for combined axial and transverse motions of two Euler–Bernoulli beams connected through two legs to a joint. We prove that the joint-leg-beam system is polynomially stable, but not exponentially stable with linear control of velocity feedback.

1. Introduction

In the past fifty years there have been numerous scientific studies in Rigidizable–Inflatable (RI) space structures and considerable progress has been made in the development of new material and technologies for the design and manufacturing of these structures. One of the most interesting application is that RI space structures could offer the efficiency in packaging during boost-to-orbit [9]. Understanding stability and damping properties of truss systems composed of these materials is essential to the successful operation of future systems.

Several proposed designs make use of rigid joints with special attachment “legs” which lead to the joint-leg-beam system considered in the paper (see Fig. 1). It has been proved in [5] that when both beams in the joint-leg-beam system are subject to Kelvin–Voigt damping, the associated semigroup is exponentially stable and analytic. Hence the energy of the system decays exponentially to zero, and the associated solution has smoothing properties. For the case in which only one of beams is subject to Kelvin–Voigt damping, the energy of the system decays polynomially with or without additional rotational damping in the joint. The thermoelastic behavior of the joint-leg-beam system was analyzed in [6] which also enjoys the exponential stability.

Since beam damping maybe be achieved by additional processing, it is of interest to know if damping applied on joints and legs is sufficient to ensure energy decay. In this paper, we show that in this case the energy

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http://dx.doi.org/10.1016/j.jmaa.2014.06.032

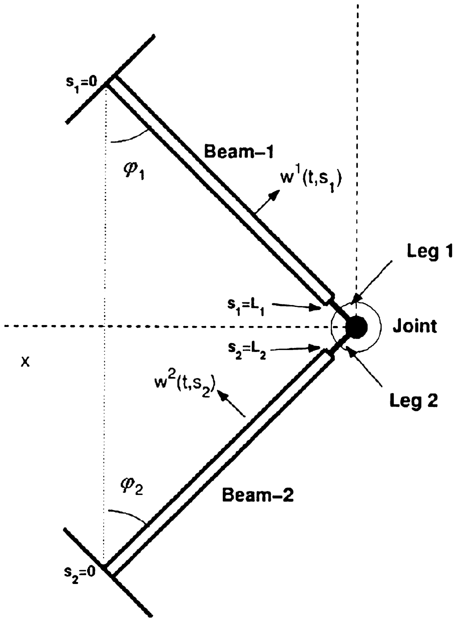

Fig. 1. Joint-leg-beam system.

of the joint-leg-beam system is not exponentially stable. Moreover, using a recent result of Borichev and Tomilov's [3] on polynomial stability characterization of bounded semigroups and the multiplier techniques, we provide precise decay estimates showing that the semigroup decays polynomially.

This paper is organized as follows. In Section 2, we show equations of motion of the joint-leg-beam system and semigroup setting of the system. Section 3 is devoted to the proof of polynomial stability of the semigroup of the joint-leg-beam system with linear control of velocity feedback.

2. Preliminaries

In this section we present some preliminary results and well-posedness of the joint-leg-beam system with feedback control on joint and legs. The motion of the closed-loop system is described as the following equations:

$$\begin{aligned}
&\rho_j A_j \frac{\partial^2}{\partial t^2} w^j(t,s_j) + E_j I_j \frac{\partial^4}{\partial s_j^4} w^j(t,s_j) = 0, \quad s_j \in (0,L_j),\ t>0,\\
&\rho_j A_j \frac{\partial^2}{\partial t^2} u^j(t,s_j) - E_j A_j \frac{\partial^2}{\partial s_j^2} u^j(t,s_j) = 0, \quad s_j \in (0,L_j),\ t>0,
\end{aligned} \tag{1}$$

$$\mathcal{M}\frac{d^2}{dt^2}\begin{pmatrix} x(t)\\ y(t)\\ \theta^1(t)\\ \theta^2(t)\end{pmatrix} = \mathcal{C}\begin{pmatrix} E_1 I_1 \frac{\partial^2}{\partial s_1^2} w^1(t,L_1)\\ E_1 I_1 \frac{\partial^3}{\partial s_1^3} w^1(t,L_1)\\ E_2 I_2 \frac{\partial^2}{\partial s_2^2} w^2(t,L_2)\\ E_2 I_2 \frac{\partial^3}{\partial s_2^3} w^2(t,L_2)\\ E_1 A_1 \frac{\partial}{\partial s_1} u^1(t,L_1)\\ E_2 A_2 \frac{\partial}{\partial s_2} u^2(t,L_2)\end{pmatrix} - \mathcal{K}\frac{d}{dt}\begin{pmatrix} x(t)\\ y(t)\\ \theta^1(t)\\ \theta^2(t)\end{pmatrix}, \quad t>0, \tag{2}$$

where $\mathcal{M}$ and $\mathcal{C}$ are 4×4 and 6×6 matrices given by