J. Math. Anal. Appl. 420 (2014) 1744–1761

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Scaling laws and the rate of convergence in thin magnetic films

Davit Harutyunyan

University of Utah, UT, United States

ARTICLE INFO

Article history:
Received 4 February 2014
Available online 1 July 2014
Submitted by A. Daniilidis

Keywords:
Thin magnetic films
Magnetic wires
Magnetization reversal
Domain wall

ABSTRACT

We study static 180 degree domain walls in thin infinite magnetic films. We establish the scaling of the minimal energy by Γ-convergence and the energy minimizer profile, which turns out to be the so-called *transverse wall* as predicted in earlier numerical and experimental work. Surprisingly, the minimal energy decays faster than the area of the film cross section at an infinitesimal cross section diameter. We establish a rate of convergence of the rescaled energies as well.

Contents

1. Introduction

1.1. Micromagnetics

In the theory of micromagnetics the energy of micromagnetics of a ferromagnetic body $\Omega \in \mathbb{R}^3$ is given by

$$E(m) = A_{ex}\int_{\Omega} |\nabla m|^2 + K_d\int_{\mathbb{R}^3} |\nabla u|^2 + Q\int_{\Omega} \varphi(m) - 2\int_{\Omega} H_{ext}\cdot m,$$

http://dx.doi.org/10.1016/j.jmaa.2014.06.056

where $m: \Omega \to \mathbb{S}^2$ with $m = 0$ in $\mathbb{R}^3 \setminus \Omega$ is a unit vector field representing the magnetization vector, A_{ex}, K_d, Q are material parameters, H_{ext} is the externally applied magnetic field, φ is the anisotropy energy density and u is the induced field potential, obtained from Maxwell's equations of magnetostatics,

$$\begin{cases} \operatorname{curl} H_{ind} = 0 & \text{in } \mathbb{R}^3, \\ \operatorname{div}(H_{ind} + m) = 0 & \text{in } \mathbb{R}^3, \end{cases}$$

where $H_{ind} = \nabla u$. Namely, u is a weak solution of

$$\Delta u = \operatorname{div} m \quad \text{in } \mathbb{R}^3,$$

i.e., ∇u is the Helmoltz projection of m onto the L^2 closure of the gradient fields in $L^2(\mathbb{R}^3)$. The energy density φ is a non-negative function called the anisotropy energy density. It is typically a polynomial, with the symmetry properties inherited from those of the underlying crystalline lattice. The zeroes of φ form the set of preferred directions of magnetization (easy axes), e.g., [6]. According to the theory of micromagnetics, stable magnetization patterns are described by the minimizers (global and local) of the micromagnetic energy functional, e.g., [14,6–8]. This is a non-convex and nonlocal minimization problem due to the non-convex constraint $|m| = 1$ in Ω. This theory is used for the analysis and design of magnetic devices. It explains observations on many length scales, and it also explains the magnetic hysteresis, through the multiplicity of local minima, e.g., [6].

1.2. Motivation

In recent years the study of thin structures in micromagnetics, in particular thin films and wires, has been of great interest, see [1,2,5,9,16–21] for nanowires and [4,7,8,10,15,17]. It was suggested in [1] that magnetic nanowires can be used as storage devices. It is known that the magnetization pattern reversal time is closely related to the writing and reading speed of such a device, thus it has been suggested to study the magnetization reversal and switching processes. In [9] the magnetizetion reversal process has been studied numerically in cobalt nanowires by the Landau–Lishitz–Gilbert equation. In thin wires the transverse mode has been observed: the magnetization in almost constant on each cross section forming a domain wall that propagates along the wire, while in relatively thick wires the vortex wall has been observed: the magnetization is approximately tangential to the boundary and forms a vortex which propagates along the wire. In [13] similar study has been done for thin nickel wires and the same results have been observed. When a homogenous external field is applied in the axial direction of the wire facing the homogenous magnetization direction, then at a critical strength the reversal of the magnetization typically starts at one end of the wire creating a domain wall, which moves along the wire. The domain wall separates the reversed and the not yet reversed parts of the wire. In [3] Cantero-Alvarez and Otto considered the problem of finding the scaling of critical field in terms of the thin film cross section and material parameters. The authors found four different scalingS and corresponding four different regimes. In Fig. 1 one can see the transverse and the vortex wall longitudinal and cross section pictures for wires with a rectangular cross section.

A distinctive crossover has been observed between the two different modes, which is expected to occur at a critical diameter of the wire. It has been suggested that the magnetization switching process can be understood by analyzing the micromagnetics energy minimization problem for different diameters of the cross section. In [16] K. Kühn studied 180 degree static domain walls in magnetic wires with circular cross sections. Kühn proved that indeed, the transverse mode must occur in thin magnetic wires as was predicted by experimental and numerical analysis before in [9] and in [13], while in thick wires a vortex wall has the optimal energy scaling. Some of the results proven by K. Kühn for thin wires has been later generalized in [12] to any wires with a bounded, Lipschitz and rotationally symmetric cross sections, see also [11]. Slastikov