J. Math. Anal. Appl. 423 (2015) 863–876

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

The mild and weak solutions of a stochastic parabolic Anderson equation

Lijun Miao[a,b,*]

[a] School of Mathematics, Jilin University, Changchun 130012, China
[b] Department of Mathematics, Baicheng Normal University, Baicheng 137000, China

ARTICLE INFO

Article history:
Received 18 March 2014
Available online 28 September 2014
Submitted by X. Zhang

Keywords:
Brownian motion
Gaussian potential
Mild solution
Weak solution

ABSTRACT

In this paper, we investigate the existence of the solutions to the stochastic parabolic Anderson equation with Gaussian potential and perturbed by external force that driven by classical Brownian motion. We show the sufficient conditions that the shape function must satisfy for the existence of the mild solution and weak solution to this type of equation.

1. Introduction

This paper is devoted to studying the existence of the solutions to the parabolic stochastic partial differential equation with random potential

$$\begin{cases} \partial_t u(t,x) = \frac{1}{2}\triangle u(t,x) + V(x)u(t,x)\dot{W}(t), \\ u(0,x) = u_0(x), \quad x \in \mathbb{R}^d, \end{cases} \tag{1}$$

where $u_0(x)$ is a bounded measurable function, $V(x)$ is a Gaussian potential, $x \in \mathbb{R}^d$, and $W(t)$ is a 1-dimensional Brownian motion. This is a type of parabolic Anderson model perturbed by $W(t)$. It is well known that the classical parabolic Anderson model is the following form of partial differential equation of parabolic type

$$\begin{cases} \partial_t u(t,x) = \kappa\triangle u(t,x) + V(t,x)u(t,x), \\ u(0,x) = u_0(x), \quad x \in \mathbb{R}^d, \end{cases}$$

* Correspondence to: School of Mathematics, Jilin University, Changchun 130012, China.
E-mail address: miaolj12@mails.jlu.edu.cn.

http://dx.doi.org/10.1016/j.jmaa.2014.09.056
0022-247X/

where $\kappa > 0$ is a diffusion constant, and $V(t, x)$ a random potential. This model appears in the context of chemical kinetics and population dynamics. Its name goes back to the work of Anderson [2] on the electrons transport in some random lattices. There are many achievements in studying the existence and uniqueness of the solution and the long time behavior and spatial intermittency of the solution (see [1,3,5,16]). As the potential $V(x)$ is Hölder continuous and bounded, Gärtner and König in [9,10] show that the solution to parabolic Anderson model exists and has the following Feynman–Kac representation

$$\begin{aligned} u(t,x) &= \mathbb{E}_x\left[\exp\left\{\int_0^t V(B_{2\kappa s})ds\right\}u_0(B_{2\kappa t})\right] \\ &= \mathbb{E}_x\left[\exp\left\{(2\kappa)^{-1}\int_0^{2\kappa t} V(B_s)ds\right\}u_0(B_{2\kappa t})\right], \quad x \in \mathbb{R}^d. \end{aligned}$$

For parabolic Anderson model, a related problem is Brownian motion in random potentials. This problem has been extensively investigated in recent two decades (see [11,14,17]), which is used to describe the trajectory of a Brownian particle that is trapped by the obstacles randomly distributed in the space. Let nonnegative $K(x)$ be a properly chosen shape function and $\omega(x)$ a Brownian sheet with mean zero on $\mathbb{R}^d$. We define the random function

$$V(x) = \int_{\mathbb{R}^d} K(y - x)\omega(dy), \tag{2}$$

which heuristically represents the total trapping energy at $x \in \mathbb{R}^d$ generated by the obstacles. Chen and Alexey studied unbounded and non-local shape functions in [6], they constructed some more physically realistic random potentials that can be consistent to Newton's law of universal attraction using renormalization method, and obtained the mild solution to related parabolic Anderson models with renormalized Poisson potential.

In our model, we investigate Brownian motion in random potential, and the system is affected by $W(t)$. That is to say, some obstacles are a family of particles distributed in the space as the centered Gaussian field $\omega(dx)$, where $\omega(dx)$ is a Gaussian random measure with $\mathbb{E}_\omega[|\omega(dx)|^2] = dx$. An additional particle executes random movement in the space $\mathbb{R}^d$ whose trajectory $B_s (s \geq 0)$ is a d-dimensional Brownian motion. $W(t)$ denotes the external random perturbation. Throughout, $\omega(dx)$, B_s and $W(s)$ are independent each other and the notations $\mathbb{E}_\omega$, $\mathbb{E}_0$ and $\mathbb{E}_W$ are used for the expectations with respect to the Gaussian field $\omega(dx)$, Brownian motion B_s and $W(s)$, respectively.

In this setting, let $K(x) \geq 0$ be a properly chosen (the conditions it should be satisfied with will be given in Theorem 1.1 and Theorem 1.2 respectively) shape function on $\mathbb{R}^d$ and let the potential (2) represents the total trapping energy at $x \in \mathbb{R}^d$ generated by the Gaussian obstacles. The random integral

$$\int_0^t V(B_s)dW(s)$$

represents the total action from both the Gaussian obstacles and external force $W(s)$ and over the Brownian particle up to the time t. We consider the parabolic Anderson model in a random potential given above.

If $u(t, x)$ has a measurable version, the mild solution of Eq. (1) is defined as the solution of the following integral equation if exists [7]