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Persistence of periodic orbits with sliding or sewing by singular perturbation

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ABSTRACT

In this paper we deal with piecewise smooth singularly perturbed systems. We study the effect of singular perturbation when the phase portrait of the reduced problem has periodic orbits with sliding or sewing points. Counter-examples are used to show that in general, only one parameter is not sufficient to ensure the persistence of periodic orbits. With an additional parameter, derived from the Sotomayor–Teixeira regularization, we get conditions which guarantee the persistence.

1. Introduction

Many systems of relevance to applications are modeled using piecewise smooth dynamical systems. The study of such systems has, in recent years, established an important frontier between Mathematics, Physics and Engineering. They appear in various situations like mechanical systems with dry friction or with impacts, in control theory, electronic, economics, medicine and biology (see for instance [3,1,13,14]). See also [2] for a general scope of the matter.

In our approach Filippov convention [8] is considered. Filippov systems are systems modeled by different smooth ODEs (ordinary differential equations) in different open domains separated by smooth discontinuity boundaries. In the simplest case the phase space is composed by two domains such that for each domain a different ODE governs the dynamics, namely

$$\dot{x} = Z(x) = \begin{cases} F(x), & \text{if } h(x) \leq 0, \\ G(x), & \text{if } h(x) \geq 0. \end{cases} \tag{1}$$

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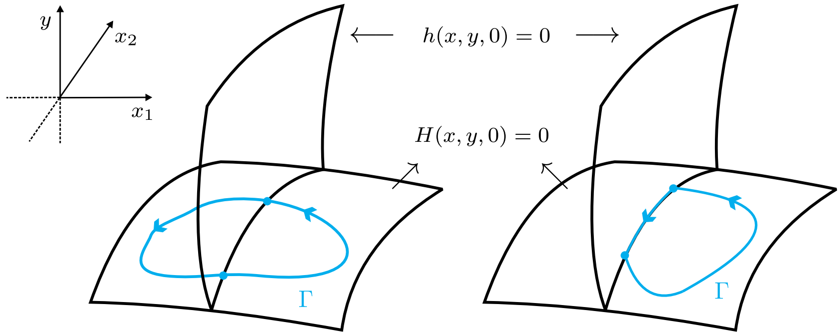

Fig. 1. Periodic orbits with sewing (left picture) and with sliding (right picture) of the reduced problem.

In Eq. (1), F and G are C^r vector fields defined on the open set $U \subset \mathbb{R}^n$, with $r \geq 1$, and $h : U \to \mathbb{R}$ is a smooth function having 0 as a regular value. The common boundary $\Sigma = \{h(x) = 0\}$ between the domains $\Sigma^- = \{h(x) \leq 0\}$ and $\Sigma^+ = \{h(x) \geq 0\}$ is called *switching manifold*.

An important question with regards to investigations of the dynamics of systems (1) is the effect of singular perturbations. For smooth dynamical systems the classical theory developed by Fenichel [7] (see also [11] and [12]) proves that any normally hyperbolic invariant manifold persists. Roughly speaking, we can say that for smooth systems any phenomenon that persists under regular perturbation also persists under singular perturbation. However, this is not the case when piecewise smooth systems are treated.

In this paper we investigate how the dynamics of piecewise smooth systems is affected by singular perturbation. For that let us consider slow–fast systems of the form

$$\dot{x} = \begin{cases} F(x, y, \varepsilon) & \text{if } h(x, y, \varepsilon) \leq 0, \\ G(x, y, \varepsilon) & \text{if } h(x, y, \varepsilon) \geq 0, \end{cases} \qquad \varepsilon \dot{y} = H(x, y, \varepsilon). \tag{2}$$

In system (2), $\varepsilon \in \mathbb{R}$ is a non-negative small parameter, $x \in \mathbb{R}^n$ and $y \in \mathbb{R}$ denote the slow and fast variables, respectively, and F, G, h and H are C^r maps which vary differentially with respect to ε, with $r \geq 1$.

Some papers have contributed to the study of this issue. For instance, in [5] the authors studied how sliding mode in Filippov systems is affected by singular perturbations. They proved that all hyperbolic equilibria and periodic orbits on the sliding region of the reduced problem (system (2) with $\varepsilon = 0$) persist for $\varepsilon \sim 0$. In [6] the effects of singular perturbations at the tangency points (i.e. points $q \in \{h(x, y, 0) = 0\} \cap \{H(x, y, 0) = 0\}$ where one of the two vector fields F or G is tangent to $\{h(x, y, 0) = 0\}$) have been analyzed. Special attention was given to fold and cusp singularities, that is when one of the vector fields F or G has a quadratic or cubic contact with $\{h(x, y, 0) = 0\}$. They proved that singularities of the kind fold are robust with respect to singular perturbations. On the other hand, cusp singularities are not robust with respect to singular perturbations. They also studied the unfolding of cusp singularities and hyperbolic equilibria.

The present work is a continuation of previous ones aforementioned. We study the effects of singular perturbations when the phase portrait of the reduced problem has periodic orbits with sewing or with sliding points. See Fig. 1 for an illustrative picture of such orbits. Here we use the term "*slide*" for both slip or escape situations.

We stress out that the robustness of periodic motion with sliding has already been addressed by some authors. In [15], Sieber and Kowalczyk considered periodic orbits with an infinitesimally small sliding segment, that is, close to a grazing-sliding bifurcation, and proved that the local return map around the grazing periodic orbit develops a discontinuity if the condition on the existence of an attracting sliding region is violated. So stable periodic motion with sliding is not robust with respect to singular perturbations. In [9, 10] the author studied this question under the simplifying assumption that the switching function does not depend on the fast variable y ($\frac{\partial h}{\partial y} = 0$ in (2)). This allowed them to prove that stable periodic orbits with sliding persist, but acquire small boundary layers after switching. As it was pointed out in [15], in practice,