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An infinite horizon stochastic maximum principle for discounted control problem with Lipschitz coefficients

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ABSTRACT

In the present work, a stochastic maximum principle for discounted control of a certain class of degenerate diffusion processes with global Lipschitz coefficient is investigated. The value function is given by a discounted performance functional, leading to a stochastic maximum principle of semi-couple forward–backward stochastic differential equation with non-smooth coefficients. The proof is based on the approximation of the Lipschitz coefficients by smooth ones and the approximation of the infinite horizon adjoint process.

1. Introduction

Stochastic optimal control has been extensively studied in the past decades due to its applications to mathematical finance, insurance, economics, engineering, etc. There are two main techniques to solve stochastic optimal control: The dynamic programming and the stochastic maximum principle. For the dynamic programming, the reader may consult [11,22] and references therein.

In this paper, we shall use stochastic maximum principle to solve an infinite horizon stochastic optimal control problem when the coefficients of the state process are non-smooth. There have been many studies on stochastic maximum principle. Under smoothness of the coefficients of the state process, Kushner [15,16] introduces the necessary stochastic maximum principle for a class of controls adapted to a fixed filtration. This work was extended to a more general setting in [4–6,14] under the assumption that the diffusion coefficient is control-free. The maximum principle is given in terms of an adjoint equation which is solution to a backward stochastic differential equation (BSDE). The previous results were extended by Peng [19] in the case of a control-dependent diffusion coefficient. The maximum principle in this case is given in terms

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of a first order and second order adjoint equations. The latter are solutions to the non-linear BSDE. Let mention also that the stochastic maximum principle was extended to system with jumps in [12].

In all the above mentioned work, it is assumed that the coefficients of the controlled process are smooth. However, it is possible to weaken the conditions on the coefficients. In [18], the author uses the Clark generalized gradient and stable convergence of probability measure to prove a finite horizon stochastic maximum principle when the coefficients are non-smooth. Using the Krylov estimate, the authors in [1] prove a finite horizon stochastic maximum principle when the coefficients are Lipschitz with the diffusion coefficient been elliptic. The previous result was extended in the case of a degenerate diffusion coefficient in [2] (see also [3,8]).

In this paper, we generalize the previous result in infinite horizon. More precisely, assuming that the state coefficients are Lipschitz (with the diffusion coefficient being degenerate), we establish an infinite horizon stochastic maximum principle for a discounted control problem. Since the value function is given by a discounted cost functional, it can be seen as a solution to a linear backward stochastic differential equation. With this observation, we also extend the above mention works on stochastic maximum principle for non-smooth coefficients, to a stochastic maximum principle for forward–backward systems. We use the technique of absolute continuity of probability measure (see [7]) to define a linearized version of the controlled process. We also defined a slightly different controlled process on an enlarged probability space with the initial condition been taken as a random variable. As for maximum principle for infinite horizon stochastic optimal control with smooth coefficients, the reader may consult [13] and [17] and the references therein.

The paper is organized as follows: In Section 2, we state the control problem and give some preliminary results. Section 3 is devoted to the study of the infinite horizon discounted control problem. This section also contains the main results of the paper. In Appendix B, we prove some needed results.

2. Statement of the problem and preliminary result

2.1. Statement of the problem

In the following, we denote by $(\Omega, \mathscr{F}, P)$ a complete probability space. Let $\mathbb{F} = \{\mathscr{F}_t\}_{0\leq t}$ be the completion of the natural filtration generated by the Brownian motion $(B(t))_{t\geq 0}$, where $\mathscr{F}_0$ contains all the P-null sets of $\mathscr{F}$ and $\mathscr{F}_\infty = \bigcup_{t\geq 0}\mathscr{F}_t$. It is a complete right continuous filtration. We denote by $|\cdot|$ and $\|\cdot\|$ the Euclidean norms in $\mathbb{R}^d$ and $\mathbb{R}^{d\times N}$, respectively. In the following, we define some space of processes.

Definition 2.1. Let $\alpha \in \mathbb{R}$, $p \geq 0$ and $\mathscr{Y}$ be a Banach space with norm $\|\cdot\|_{\mathbb{Y}}$

- $L^p_{\mathbb{R}_+,\alpha}(\mathscr{Y})$ is the space of all $\mathscr{F}_t$-measurable random variables $X : \Omega\times\mathbb{R}_+ \to \mathscr{Y}$ such that

$$E\left[e^{\alpha t}\|X(t)\|^p_{\mathscr{Y}}\right] < \infty,$$

- $S^p_{\mathbb{R}_+,\alpha}(\mathscr{Y})$ is the space of all càdlàg, adapted processes $X : \Omega\times\mathbb{R}_+ \to \mathscr{Y}$ such that

$$E\left[\sup_{0\leq t} e^{\alpha t}\|X(t)\|^p_{\mathscr{Y}}\right] < \infty,$$

- $H^p_{\mathbb{R}_+,\alpha}(\mathscr{Y})$ is the space of all predictable processes $X : \Omega\times\mathbb{R}_+ \to \mathscr{Y}$ such that

$$E\left[\int_0^\infty e^{\alpha t}\|X(t)\|^p_{\mathscr{Y}}\,\mathrm{d}t\right] < \infty,$$

- we define $\mathscr{V}_\alpha = S^p_{\mathbb{R}_+,\alpha}(\mathscr{Y}) \times H^p_{\mathbb{R}_+,\alpha}(\mathscr{Y})$.