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Further investigations into the stability and bifurcation of a discrete predator–prey model

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ABSTRACT

In this paper we revisit a discrete predator–prey model with a non-monotonic functional response, originally presented in Hu, Teng, and Zhang (2011) [6]. First, by citing several examples to illustrate the limitations and errors of the local stability of the equilibrium points E_3 and E_4 obtained in this article, we formulate an easily verified and complete discrimination criterion for the local stability of the two equilibria. Here, we present a very useful lemma, which is a corrected version of a known result, and a key tool in studying the local stability and bifurcation of an equilibrium point in a given system. We then study the stability and bifurcation for the equilibrium point E_1 of this system, which has not been considered in any known literature. Unlike known results that present a large number of mathematical formulae that are not easily verified, we formulate easily verified sufficient conditions for flip bifurcation and fold bifurcation, which are explicitly expressed by the coefficient of the system. The center manifold theory and Project Method are the main tools in the analysis of bifurcations. The theoretical results obtained are further illustrated by numerical simulations.

1. Introduction

Over the last two decades, increasing numbers of studies have investigated discrete population models. See, for example, [1–7,9,11,12,14,15] and the references cited therein. This is due in part to the fact that the discrete models are more appropriate to describe the evolution rule of population than the continuous models, when populations have non-overlapping generations or the populations size is small. Additionally, some discrete models have more richer dynamical behavior than the corresponding continuous models. For example, it is well-known that the single-species discrete Logistic model has very complex dynamical behavior, from period-doubling bifurcations to chaotic behavior [8,13], whereas the solutions for corresponding continuous Logistic model are monotonic.

Based on the following continuous-time predator–prey model considered by Ruan and Xiao in [14],

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$$\begin{cases} \dot{x} = rx\left(1 - \dfrac{x}{K}\right) - \dfrac{xy}{a + x^2}, \\ \dot{y} = y\left(\dfrac{\mu x}{a + x^2} - D\right), \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ denote the numbers of prey and predator at time t, respectively, $K > 0$ is the carrying capacity of the prey, $r > 0$ is the intrinsic growth rate, $\mu > 0$ is the conversion coefficient, $D > 0$ is the death rate of the predator and $a > 0$ is the half-saturation constant, in 2011. Hu, et al. [6], considered the stability and bifurcation of the following discrete model, corresponding to (1.1),

$$\begin{cases} x(n+1) = x(n)\exp\left[r\left(1 - \frac{x(n)}{K}\right) - \frac{y(n)}{a+x^2(n)}\right], \\ y(n+1) = y(n)\exp\left[\frac{\mu x(n)}{a+x^2(n)} - D\right], \end{cases} \tag{1.2}$$

where r, a, μ, D and K are defined as in model (1.1). It is assumed that the initial values of the solutions in system (1.2) satisfy $x(0) > 0$, $y(0) > 0$ and all the parameters are positive. It is easy to prove that if the initial values $(x(0), y(0))$ of the system (1.2) are positive, the corresponding solution $(x(n), y(n))$ is positive too.

In view of the biological meaning of system (1.2), one only considers the stability and bifurcation for nonnegative equilibria of the system.

For the existence of the nonnegative equilibria of system (1.2), the following results may be easily derived.

Proposition 1.1. *The following conclusions are true for the existence of the nonnegative equilibria of system (1.2).*

(i) $\mu^2 - 4aD^2 < 0$. *System (1.2) has only two nonnegative equilibria* $E_0(0, 0)$ *and* $E_1(K, 0)$.

(ii) $\mu^2 - 4aD^2 = 0$. *If* $0 < K \le \frac{\mu}{2D}$, *then system (1.2) has only two nonnegative equilibria* $E_0(0, 0)$ *and* $E_1(K, 0)$; *if* $\frac{\mu}{2D} < K$, *then system (1.2) has three nonnegative equilibria:* $E_0(0, 0)$, $E_1(K, 0)$ *and one positive equilibrium point* $E_2(x_2, y_2)$, *where,*

$$x_2 = \frac{\mu}{2D}, \qquad y_2 = r\left(a + x_2^2\right)\left(1 - \frac{x_2}{K}\right).$$

(iii) $\mu^2 - 4aD^2 > 0$. *If* $0 < K \le \frac{\mu - \sqrt{\mu^2 - 4aD^2}}{2D}$, *then system (1.2) has only two boundary equilibria,* $E_0(0, 0)$ *and* $E_1(K, 0)$. *If*

$$\frac{\mu - \sqrt{\mu^2 - 4aD^2}}{2D} < K \le \frac{\mu + \sqrt{\mu^2 - 4aD^2}}{2D},$$

then, system (1.2) has three nonnegative equilibria: $E_0(0, 0)$, $E_1(K, 0)$ *and one positive equilibrium point* $E_3(x_3, y_3)$, *where*

$$x_3 = \frac{\mu - \sqrt{\mu^2 - 4aD^2}}{2D}, \qquad y_3 = r\left(a + x_3^2\right)\left(1 - \frac{x_3}{K}\right).$$

If $K > \frac{\mu + \sqrt{\mu^2 - 4aD^2}}{2D}$, *then system (1.2) has four nonnegative equilibria:* $E_0(0, 0)$, $E_1(K, 0)$ *and two positive equilibria* $E_3(x_3, y_3)$, $E_4(x_4, y_4)$, *where,*

$$x_4 = \frac{\mu + \sqrt{\mu^2 - 4aD^2}}{2D}, \qquad y_4 = r\left(a + x_4^2\right)\left(1 - \frac{x_4}{K}\right).$$