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Dynamics in a diffusive predator–prey system with strong Allee effect and Ivlev-type functional response ☆

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ABSTRACT

The dynamics of a kind of reaction–diffusion predator–prey system with strong Allee effect in the prey population is considered. We prove the existence and uniqueness of the solution and give a priori bound. Hopf bifurcation and steady state bifurcation are studied. Results show that the Allee effect has significant impact on the dynamics.

1. Introduction

Predator–prey interaction is one of the basic interspecies relations for ecological and social models, and many researchers have studied the following 2-component reaction–diffusion systems, due originally to [18, 20,27]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \delta_1 \Delta u + ru\left(1 - \dfrac{u}{K}\right) - pvh(ku), \\ \dfrac{\partial v}{\partial t} = \delta_2 \Delta v + qvh(ku) - sv, \end{cases} \tag{1.1}$$

where $u(\vec{x}, t)$ and $v(\vec{x}, t)$ are the population densities of prey and predators at time t and (vector) position $\vec{x}$. The $\triangle$ is the usual Laplacian operator in $d \leq 3$ space dimensions and the parameters δ_1, δ_2, r, K, p, q, k and s are strictly positive. The 'functional response' $h(\cdot)$ is assumed to be a C^2 function.

In this paper, we consider Ivlev-type functional response, introduced by Ivlev [12], that is

$$h(\eta) = 1 - e^{-\eta}.$$

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The so-called Ivlev-type functional response was classified to the Holling-II functional response by Garvie. As in [8], we translate Eqs. (1.1) to nondimensional form:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u + u(1-u) - v\big(1-e^{-\gamma u}\big), & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} = \delta \Delta v + \beta v\big(\alpha - 1 - \alpha e^{-\gamma u}\big), & x \in \Omega,\ t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x) \geq 0, \quad v(x,0) = v_0(x) \geq 0, & x \in \Omega. \end{cases} \tag{1.2}$$

Both ecologists and mathematicians are interested in the Ivlev-type predator–prey model and much progress has been seen in the study of model (1.2), for example see [8,14,15,18,23,26,29,28,32,34,35,39,40].

Recently, Allee effect in the growth of a population has been studied extensively. Allee effect introduced by ecologist Allee [1] is such a phenomenon that the population will become extinct if its density is below certain threshold value [3,30,33,37]. Population with small density will have difficulties in finding mates, reproductive facilitation, predation, environment conditioning and inbreeding depression [6,7]. Many scholars have studied the Allee effect for its big impact on population dynamics [2,6,21,25,30,36]. It is widely accepted that the Allee effect may increase the extinction risk of low-density populations [7,16]. Therefore the population ecology investigation of the Allee effect is important to conservation biology [4,6,31].

In this paper, we assume that the functional response is of Ivlev-type [12], and similarly to [17,38], consider the prey growth rate with strong Allee effect:

$$u(1-u)\left(\frac{u}{b} - 1\right),$$

where b quantifies the intensity of the strong Allee effect with $0 < b < 1$. Therefore we have the following system:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u + u(1-u)\left(\dfrac{u}{b} - 1\right) - v\big(1-e^{-\gamma u}\big), & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} = \delta \Delta v + \beta v\big(\alpha - 1 - \alpha e^{-\gamma u}\big), & x \in \Omega,\ t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x) \geq 0, \quad v(x,0) = v_0(x) \geq 0, & x \in \Omega, \end{cases} \tag{1.3}$$

where $0 < b < 1$ represents the Allee threshold of prey, $\Omega \subset \mathbb{R}^n$ is a smooth bounded spatial domain for $n \geq 1$ and we impose a no-flux boundary condition, so it is a closed ecosystem.

The rest of this paper is structured in the following way. In Section 2, we analyze the existence and uniqueness of solutions and give a priori bound. In Section 3, we consider the stability of constant steady state solutions. In Section 4, we show the existence of time-periodic orbits with a careful Hopf bifurcation and steady state bifurcation analysis. We present some numerical simulations to illustrate our theoretical analysis. In this paper, we denote by $\mathbb{N}_0$ the set of all the nonnegative integers and $\mathbb{R}^+$ the set of all the positive real numbers.

2. Basic dynamics

In the section, we study the existence and uniqueness of solution of the system (1.3) and estimate the solution.

As in [38], with method of the lower-solution and upper-solution (see [24,22,41]), and some results of asymptotically autonomous (see [5,9,13,19]), for system (1.3), we have the following results and here omit the proof.