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Models for determining how many natural enemies to release inoculatively in combinations of biological and chemical control with pesticide resistance

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A R T I C L E I N F O

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Combining biological and chemical control has been an efficient strategy to combat the evolution of pesticide resistance. Continuous releases of natural enemies could reduce the impact of a pesticide on them and the number to be released should be adapted to the development of pesticide resistance. To provide some insights towards this adaptation strategy, we developed a novel pest–natural enemy model considering both resistance development and inoculative releases of natural enemies. Three releasing functions which ensure the extinction of the pest population are proposed and their corresponding threshold conditions obtained. Aiming to eradicate the pest population, an analytic formula for the number of natural enemies to be released was obtained for each of the three different releasing functions, with emphasis on their biological implications. The results can assist in the design of appropriate control strategies and decision-making in pest management.

1. Introduction

Chemical pest control is defined as the reduction of a pest population by using chemical pesticides. Because the latter are relatively cheap and are readily available, chemical pest control is the most common method used. However, with regular and repeated spraying, a pest may develop resistance to the pesticide quickly. As a consequence, there has been decreased susceptibility of pest populations to pesticides that were previously effective [23]. Studies indicate that more than 500 species of pests have now developed resistance to some pesticides [10,15,34]. Pesticide resistance also leads to increases in farmers' losses, even

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though more pesticides are used. For example, in the USA, farmers lost 7% of their crops to pests in the 1940s, but since the 1980s the percentage lost has increased to 13%; nevertheless, even more pesticides are still being used [35].

Therefore, in order to reduce or delay the development of pesticide resistance, a number of strategies have been proposed including pesticide switching or rotation, avoiding unnecessary pesticide applications, leaving untreated refuges where susceptible pests can survive, and using non-chemical control techniques [7]. The concept of integrated pest management (IPM) [9,21,38,36,37,24], an integrated combination of more than one method (such as biological control, chemical control, cultural tactics, breeding for host-plant resistance etc.), has been developed aiming to maintaining the density of pest populations below their Economic Injury Levels (EIL).

Pesticide switching or rotation is the main and usual method to fight pesticide resistance. In our recent study [16], we developed a pest population growth model incorporating the evolution of pesticide resistance, and introduced three different pesticide switching methods: threshold condition-guided, density-guided and EIL-guided. For each method, we discussed the optimal switching time. Moreover, we compared these three methods, and our results indicated that either the density-guided method or the EIL-guided method was the optimal pesticide switching method, depending on the frequency (or period) of pesticide applications.

Although pesticide switching is an efficient pest control method, it may initiate multi-pesticide resistance against which IPM is proposed. Biological control is often a key component of such an IPM strategy [11,22,27], and is the key method for responding to pesticide resistance [25]. Biological control aims to reduce pest populations by releasing natural enemies at some critical time when insufficient reproduction of released natural enemies is likely to occur and pest control will be achieved exclusively by the released individuals themselves [13,20]. This approach is known as augmentation of natural enemies. There are two general means to augmentation: inundative releases and inoculative releases [25].

Inundative release, the releasing of large numbers of natural enemies for immediate reduction of a damaging or near-damaging pest population, has been used frequently. Examples include the mass release of the egg parasite *Trichogramma* for controlling the eggs of various types of moths [25]. This approach is usually implemented impulsively, and it has been widely studied through mathematical models, especially with impulsive differential equations [20,29,30,28,31–33,17]. For example, Liang et al. [17] developed two impulsive pest–natural enemy interaction models with the development of pesticide resistance in which pulsed actions such as pesticide applications and natural enemy releases were considered. A goal of our present paper is to estimate how the number of natural enemies to be released should be changed with increasing pesticide resistance, to ensure pest eradication. Our analytic study shows how to change this according to the cumulative number of dead natural enemies before the next scheduled release time.

Inoculative releasing refers to the continuous releasing of small numbers of natural enemies at prescribed intervals throughout the pest period, starting when the pest population is very low. Examples include the release of predatory mites to protect greenhouse crops, and the inoculation of soils with the milky spore pathogen (*Bacillus popillae*) to control Japanese beetle grubs [18,25]. This releasing strategy is also known as consecutive release.

So, if the pests develop resistance to the pesticide and biological control is implemented by inoculative release, how many natural enemies need to be released as the resistance to the pesticide evolves? To answer such questions, we develop a hybrid impulsive pest–natural enemy model with pesticide sprays in pulses, evolution of pesticide resistance and inoculative releases of natural enemies. Three different releasing methods are proposed and considered: (a) fixed numbers of natural enemies released over the period of pesticide application; (b) the numbers of natural enemies released is linearly dependent on time; (c) the numbers of natural enemies released is exponentially dependent on time. For each releasing method, we investigate the threshold condition for pest eradication, and the optimal number of natural enemies to be released at a particular time or at prescribed intervals.