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Application of a biparametric perturbation method to large-deflection circular plate problems with a bimodular effect under combined loads

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ABSTRACT

The large deflection condition of a bimodular plate may yield a dual nonlinear problem where the superposition theorem is inapplicable. In this study, the bimodular Föppl–von Kármán equations of a plate subjected to the combined action of a uniformly distributed load and a centrally concentrated force are solved using a biparametric perturbation method. First, the deflection and radial membrane stress were expanded in double power series with respect to the two types of loads. However, the biparametric perturbation solution obtained exhibited a relatively slow rate of convergence. Next, by introducing a generalized load and its corresponding generalized displacement, the solution is expanded in a single power series with respect to the generalized displacement parameter, thereby leading to the better convergence on the solution. A numerical simulation is also used to verify the correctness of the biparametric perturbation solution. The introduction of a bimodular effect will modify the stiffness of the plate to some extent. In particular, the bearing capacity of the plate will be strengthened further when the compressive modulus is greater than the tensile modulus.

1. Introduction

Flexible plate-like structures constructed from advanced materials have many applications in engineering practices because they feature large deformation and apparently different mechanical responses in their tension and compression states due to external loads. During their theoretical analysis, it is necessary to consider the geometrical nonlinearity (large deformation) and the nonlinearity of materials (different elastic properties in terms of tension and compression) to facilitate a better mechanical characterization.

In reality, all materials may exhibit different elastic properties in terms of tension and compression, but these characteristics are often neglected due to the complexity of their analysis. Materials that have

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apparently different moduli are known as bimodular materials [16,17], for example, ceramics, graphite, concrete, and some composites. During recent decades, many studies have described useful material models for studying bimodular materials. In particular, Ambartsumyan's bimodular model [1,2] for isotropic materials has attracted the most attention in the engineering community. This model assesses different moduli in terms of tension and compression based on the positive–negative signs of principal stresses, which is especially important for the analysis and design of structures. In structural engineering, an engineer often requires a comprehensive understanding of the potential development of cracks, which are due mainly to increases in the principal tensile stress in a beam or plate member that comprises the structure. With the exceptions of some fundamental problems, however, acquiring the states of the stresses in a structure relies only upon finite element modeling analysis based on an iterative strategy [8,21,23–25].

In addition to the bimodular effect in materials, as mentioned above, the large deflection problem that affects thin plates also needs to be addressed. The well-known classical Föppl–von Kármán equations comprise two nonlinear high-order partial differential equations that consider two types of deformation: bending and tension. Various methods based on either analytical approaches or numerical techniques have been developed to solve the classical von Kármán equations, but a valid analytical solution that is applicable for convenient structural analysis is still unavailable.

Among these analytical methods, perturbation methods based on a certain small parameter may achieve this aim. For example, Poincaré's perturbation method is a representative analytical method that is capable of solving nonlinear problems in practical applications. This method provides the solution for an initial or boundary value problem in the form of an asymptotic series with respect to a certain parameter. This parameter either appears explicitly or is introduced artificially into the problem. In brief, the typical process employed to solve von Kármán equations for thin plates using perturbation methods aims to expand the solution in the form of ascending powers with respect to a known parameter (e.g., load or deflection). The unknown functions in the solution may be determined subsequently by substituting the asymptotic expansion into the governing equations and corresponding boundary conditions, before decomposing them.

In applications of the perturbation method, the choice of perturbation parameter is a key problem because the correct choice will lead directly to the asymptotic solution with better convergence. In general, there are two basic choices of perturbation parameter: load and displacement. Vincent [22] first used the external load as a perturbation parameter to solve von Kármán equations, but the convergence of the solution obtained was relatively poor. Chien [5] utilized the central deflection as a perturbation parameter to solve the same problem and obtained better convergence where the solution agreed well with the experimental results. In addition to the load and central deflection, several other perturbation parameters are possible, e.g., a generalized displacement [14], a linear function of Poisson's ratio [20], and an average angular deflection [15]. Chen and Kuang [4] discussed the differences among the possible parameters and proposed that the variation principle may be used to solve the large deflection problem with a general perturbation parameter.

Under a combined load, the perturbation method based on central deflection proposed by Chien [5] will encounter difficulties because the combined loads cannot be expanded with respect to the central deflection. Moreover, the method proposed by Vincent [22] that uses a singular load as perturbation parameter appears to be inappropriate because the problem we address is concerned with two types of load: a uniformly distributed load and a concentrated load. Therefore, the so-called biparametric perturbation method is associated with the solution of this problem, although there have been relatively few studies in this field. Nowinski and Ismail [18] used a multi-parameter perturbation method to solve the problem of plates. Chien [6] used an iterative method to solve the nonlinear arc length problem in the design and construction of Yongjiang Railway Bridge in Ningbo, which was the first application of a multi-parameter perturbation method to solving beam problems. He and Chen [7] obtained a biparametric perturbation solution for the same practical problem by simplifying the governing equation. More recently, He et al. [12] further illustrated