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Complete convergence for moving average processes associated to heavy-tailed distributions and applications

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ABSTRACT

The complete convergence is obtained for the moving average processes associated to heavy-tailed distributions via integral test. As the applications, two versions of Chover's law of the iterated logarithm are deduced.

1. Introduction and main results

The concept of complete convergence was first introduced by Hsu and Robbins [13] as follows. A sequence of random variables $\{U_n, n \geq 1\}$ is said to converge completely to a constant C if $\sum_{n=1}^{\infty} P\{|U_n - C| > \varepsilon\} < \infty$ for all $\varepsilon > 0$. By the Borel–Cantelli lemma, this implies $U_n \to C$ almost surely and the converse implication is not necessarily true if $\{U_n, n \geq 1\}$ are not independent. Hsu and Robbins [13] proved that the sequence of arithmetic means of independent and identically distributed random variables converges completely to the expected value if the variance of the summands is finite. And Erdös [9,10] proved that the converse is also true.

This result has been generalized and extended in several directions, for example, see Katz [18], Baum and Katz [2], Bai and Su [1], Gut [12], Hu et al. [15], etc. It is worthwhile to point that Katz [18], Baum and Katz [2] obtained the following results: if $0 < p < 2$ and $r \geq 1$ then $E|X|^{rp} < \infty$ if and only if

$$\sum_{n=1}^{\infty} n^{r-2} P\left\{\left|\sum_{j=1}^{n} X_j - nb\right| > \varepsilon n^{1/p}\right\} < \infty \quad \text{for all } \varepsilon > 0, \tag{1.1}$$

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if and only if

$$\sum_{n=1}^{\infty} n^{r-2} P\left\{ \max_{1 \le k \le n} \left| \sum_{j=1}^{k} X_j - kb \right| > \varepsilon n^{1/p} \right\} < \infty \quad \text{for all } \varepsilon > 0, \tag{1.2}$$

where $b = EX$ if $rp \ge 1$ and $b = 0$ if $0 < rp < 1$.

Recently the rate of complete convergence for sequences of dependent random variables has attracted lots of attention. One of those investigations is to investigate the rate of complete convergence for moving average processes based on the independent, identically distributed random variables. The concept of the moving average processes and relevant limit results is stated in the following:

Assume that $\{X_i, -\infty < i < +\infty\}$ is a doubly infinite sequence of identically distributed random variables. Let $\{a_i, -\infty < i < +\infty\}$ be a sequence of real numbers with

$$\sum_{i=-\infty}^{\infty} |a_i|^{\delta} < \infty \tag{1.3}$$

for some $0 < \delta \le 1$ and define the moving average process as

$$Y_n = \sum_{i=-\infty}^{\infty} a_i X_{i+n}, \quad n \ge 1. \tag{1.4}$$

When $\{X, X_i, -\infty < i < +\infty\}$ is a sequence of independent and identically distributed random variables, many limiting results have been obtained for the moving average process $\{Y_n, n \ge 1\}$. For example, Ibragimov [17] established the central limit theorem, Burton and Dehling [4] obtained a large deviation principle assuming $E\exp\{tX\} < \infty$ for all t, and Li et al. [20] obtained the complete convergence result for $\{Y_n, n \ge 1\}$. All those show that the partial sums of $\{Y_n, n \ge 1\}$ have similar limiting behavior properties in comparison with the limiting properties of independent and identically distributed random variables.

For example, Hsu–Robbins result was extended by Li et al. [20] for moving average processes.

Theorem A. *Let $\{X, X_i, -\infty < i < \infty\}$ be a sequence of independent and identically distributed random variables with $EX = 0$ and $EX^2 < \infty$, $\{a_i, -\infty < i < +\infty\}$ be a sequence of real numbers satisfying (1.3) for $\delta = 1$. Suppose $\{Y_n, n \ge 1\}$ is the moving average processes defined as (1.4). Then $\sum_{n=1}^{\infty} P\{|\sum_{j=1}^{n} Y_j| > \varepsilon n\} < \infty$ for all $\varepsilon > 0$.*

Using a method different from that in Li et al. [20], Chen et al. [6] obtained the complete convergence for the maximum sums, which extended the results in Katz [18], Baum and Katz [2] partly to moving average processes.

Theorem B. *Let $\{X, X_i, -\infty < i < \infty\}$ be a sequence of independent and identically distributed random variables with $EX = 0$ and $E|X|^{rp} < 0$ for $r \ge 1$, $1 \le p < 2$, and let $\{a_i, -\infty < i < +\infty\}$ be a sequence of real numbers satisfying (1.3) for $\delta = 1$ if $rp > 1$ and $0 < \delta < 1$ if $rp = 1$. Suppose $\{Y_n, n \ge 1\}$ is the moving average process defined as (1.4). Then*

$$\sum_{n=1}^{\infty} n^{r-2} P\left\{ \max_{1 \le k \le n} \left| \sum_{j=1}^{k} Y_j \right| > \varepsilon n^{1/p} \right\} < \infty \quad \textit{for all } \varepsilon > 0.$$