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Existence results for Hughes' model for pedestrian flows

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ABSTRACT

In this paper we prove two global existence results for Hughes' model for pedestrian flows under assumptions that ensure that the traces of the solutions along the turning curve are zero for all positive times.

1. Introduction

In this paper we study the one-dimensional version of Hughes' model [15] for pedestrian flows

$$\partial_t \rho - \partial_x \left[\rho v(\rho) \frac{\partial_x \varphi}{|\partial_x \varphi|} \right] = 0, \quad |\partial_x \varphi| = c(\rho), \tag{1}$$

in the spatial domain $\Omega =]-1, 1[$, together with homogeneous Dirichlet boundary conditions

$$\rho(t, -1) = \rho(t, 1) = 0, \qquad \varphi(t, -1) = \varphi(t, 1) = 0, \quad t > 0 \tag{2a}$$

and initial datum

$$\rho(0, x) = \bar{\rho}(x), \quad x \in]-1, 1[. \tag{2b}$$

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Here $x \in \Omega$ is the space variable, $t \geq 0$ is the time, $\rho = \rho(t, x) \in [0, 1]$ is the (normalized) crowd density, while

$$v(\rho) = 1 - \rho, \qquad c(\rho) = 1/v(\rho)$$

are respectively the mean (normalized) velocity and the running cost. We denote

$$f(\rho) = \rho v(\rho) = \rho(1 - \rho).$$

The initial datum $\bar{\rho}$ is assumed to be in $\mathbf{L}^\infty(\Omega; \mathbb{R})$ with $\|\bar{\rho}\|_\infty < 1$. This assumption, together with the maximum principle proved in [12], will ensure that the cost (which is singular for $\rho = 1$) computed along any solution of (1), (2) is well defined.

As already observed in [2,12], the system (1) can be rewritten as

$$\partial_t \rho + \partial_x F(t, x, \rho) = 0, \qquad F(t, x, \rho) = \operatorname{sgn}\big(x - \xi(t)\big) f(\rho), \tag{3a}$$

$$\int_{-1}^{\xi(t)} c\big(\rho(t, x)\big)\, \mathrm{d}x = \int_{\xi(t)}^{1} c\big(\rho(t, x)\big)\, \mathrm{d}x. \tag{3b}$$

Indeed, in order to have a unique viscosity solution to the Dirichlet problem for φ, the derivative $\partial_x \varphi$ can change its sign just once from positive to negative. It is therefore defined the so called *turning curve* $x = \xi(t)$, where $\varphi(t, \cdot)$ reaches its maximum point. After integration of the second equation in (1), the relation (3b) states the continuity of $\varphi(t, \cdot)$ at $x = \xi(t)$, and defines it implicitly. Notice that the flux F is possibly discontinuous along $x = \xi(t)$.

Definition 1. (See [12].) A map $(t, x) \mapsto \rho(t, x)$ is an entropy weak solution of the initial–boundary value problem (2), (3) if is in $\mathbf{C^0}([0, +\infty[; \mathbf{L^1}(\Omega; [0, 1]))$ and for any $\kappa \in [0, 1]$ and any test function $\psi \in \mathbf{C_c^\infty}(\mathbb{R}^2; [0, +\infty[)$ it satisfies

$$\int_0^{+\infty} \int_{-1}^{1} \big[|\rho - \kappa| \partial_t \psi + \mathcal{F}(t, x, \rho, \kappa) \partial_x \psi\big]\, \mathrm{d}x\, \mathrm{d}t + \int_{-1}^{1} \big|\bar{\rho}(x) - \kappa\big| \psi(0, x)\, \mathrm{d}x \tag{4a}$$

$$+ \int_0^{+\infty} \big[f\big(\rho(t, -1+)\big) - f(\kappa)\big] \psi(t, -1)\, \mathrm{d}t + \int_0^{+\infty} \big[f\big(\rho(t, 1-)\big) - f(\kappa)\big] \psi(t, 1)\, \mathrm{d}t \tag{4b}$$

$$+ 2 \int_0^{+\infty} f(\kappa) \psi\big(t, \xi(t)\big)\, \mathrm{d}t \geq 0 \tag{4c}$$

where

$$\mathcal{F}(t, x, \rho, \kappa) = \operatorname{sgn}(\rho - \kappa)\big[F(t, x, \rho) - F(t, x, \kappa)\big].$$

The first line (4a) originates from the Kružkov definition of entropy weak solution in the case of a Cauchy problem, [16]. Line (4b) comes from the boundary condition introduced by Bardos et al. in [6], see also [4,8,9,17]. The latter line (4c) accounts for the discontinuity of the flux along the turning curve, see [1,3,5,13,19].

Observe that the strong traces of the solution at the boundary points exist due to the genuine non-linearity of the flux [18,20] and must satisfy