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Global existence for weak solutions of the Cauchy problem in a model of radiation hydrodynamics

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ABSTRACT

We consider a model based on the Navier–Stokes–Fourier system coupled to a transport equation, recently proposed in order to describe the thermal effects in low Mach number radiative flows. We establish global-in-time existence in weighted spaces for the associated Cauchy problem in the framework of weak solutions.

1. Introduction

We consider a simplified mathematical model of radiative flow recently introduced by Teleaga and coauthors [36,9,35]. The motion of the fluid is described by standard fluid mechanics giving the evolution of the mass density $\varrho = \varrho(t,x)$, the velocity field $\vec{u} = \vec{u}(t,x)$, and the absolute temperature $\vartheta = \vartheta(t,x)$, for $t \geq 0$ and $x \in \mathbb{R}^3$. The dynamics of radiation is described by a transport equation for the radiative intensity $I = I(t,x,\vec{\omega},\nu)$ which depends on the direction vector $\vec{\omega} \in \mathcal{S}^2$, where $\mathcal{S}^2 \subset \mathbb{R}^3$ denotes the unit sphere, and the frequency $\nu \geq 0$. The coupled system to be studied reads as follows, for $(t,x,\nu,\omega) \in (0,T) \times \mathbb{R}^3 \times (0,\infty) \times \mathcal{S}^2$

$$\partial_t \varrho + \operatorname{div}_x(\varrho\vec{u}) = 0, \tag{1.1}$$

$$\partial_t(\varrho\vec{u}) + \operatorname{div}_x(\varrho\vec{u} \otimes \vec{u}) + \nabla_x p = \operatorname{div}_x \mathbb{T}, \tag{1.2}$$

$$\partial_t\left(\varrho\left(\frac{1}{2}|\vec{u}|^2 + e\right)\right) + \operatorname{div}_x\left(\varrho\left(\frac{1}{2}|\vec{u}|^2 + e\right)\vec{u} + \vec{q} - \mathbb{T}\vec{u}\right) = -S_E, \tag{1.3}$$

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$$\frac{1}{c}\partial_t I + \vec{\omega}\cdot\nabla_x I = S. \tag{1.4}$$

Observe in this system that, contrary to the model studied in [11], radiation does not appear in the momentum equation. A motivation for this approximation is to check the non-dimensional version [5] that this term is small.

The pressure $p = p(\varrho, \vartheta)$ and the specific internal energy $e = e(\varrho, \vartheta)$ are related through Maxwell's equation

$$\frac{\partial e}{\partial \varrho} = \frac{1}{\varrho^2}\left(p(\varrho, \vartheta) - \vartheta\frac{\partial p}{\partial \vartheta}\right). \tag{1.5}$$

Furthermore, $\mathbb{T}$ is the stress tensor determined by Newton's rheological law

$$\mathbb{T} = \mu\left(\nabla_x\vec{u} + \nabla_x^t\vec{u} - \frac{2}{3}\operatorname{div}_x\vec{u}\right) + \eta\operatorname{div}_x\vec{u}\,\mathbb{I}, \tag{1.6}$$

where the shear viscosity coefficient $\mu = \mu(\vartheta) > 0$ and the bulk viscosity coefficient $\eta = \eta(\vartheta) \geq 0$ are effective functions of ϑ. Similarly, $\vec{q}$ is the heat flux given by Fourier's law $\vec{q} = -\kappa\nabla_x\vartheta$, with heat conductivity coefficient $\kappa = \kappa(\vartheta) > 0$. Finally the source S is decomposed as $S := S_{a,e} + S_s$, where

$$S_{a,e} = \sigma_a\big(B(\nu, \vartheta) - I\big), \qquad S_s = \sigma_s\left(\frac{1}{4\pi}\int_{\mathcal{S}^2} I(\cdot, \vec{\omega})\,\mathrm{d}\vec{\omega} - I\right), \tag{1.7}$$

are respectively the absorption–emission and scattering contributions and the radiative energy source in (1.3) is

$$S_E = \int_{\mathcal{S}^2}\int_0^\infty S(\cdot, \nu, \vec{\omega})\,\mathrm{d}\nu\,\mathrm{d}\vec{\omega}, \tag{1.8}$$

with the absorption coefficient $\sigma_a = \sigma_s(\nu, \vartheta) \geq 0$, and the scattering coefficient $\sigma_s = \sigma_s(\nu, \vartheta) \geq 0$. More restrictions on the structural properties of constitutive relations will be imposed in Section 2 below.

System (1.1)–(1.8) is a simplified model of radiation hydrodynamics, the physical foundations of which were described by Pomraning [32] and Mihalas and Weibel-Mihalas [27].

From a mathematical point of view, the main interest of system (1.1)–(1.4) is the fact that it mixes hyperbolic and parabolic-type equations, coupled to a non-linear integro-differential transport equation of Boltzmann's type so the global well-posedness of such a system is not clear at first glance. From a physical viewpoint, system (1.1)–(1.4) has been introduced by Teleaga and coauthors in a series of papers to simulate thermal flows modeling fires in vehicle tunnels [35,36]. Related systems have also been investigated more recently by Lowrie, Morel and Hittinger [26], Buet and Després [5], with a special attention to asymptotic regimes. Concerning existence of solutions, global existence of weak solutions for large data has been proved in a bounded domain in [11] for the "complete system" (when an extra radiative source appears in the right-hand side of (1.2)), and global existence of a unique strong solution for small data has recently been proved for the Cauchy problem in [15].

Let us mention for completeness that existence of local-in-time solutions in the inviscid case was obtained by Zhong and Jiang [38] and that a number of results in one-dimensional geometry are available (see [1, 12–14] and references therein).

Our goal in the present paper is to show that the existence theory developed in [11] can be adapted to problem (1.1)–(1.8) in the whole space $\mathbb{R}^3$, extending to large data result of [15].