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Strongly damped wave equation with exponential nonlinearities

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A B S T R A C T

In this paper, we study the initial boundary value problem for two-dimensional strongly damped wave equation with exponentially growing source and damping terms. We first show the well-posedness of this problem and then prove the existence of the global attractor in $(H_0^1(\Omega) \cap L^\infty(\Omega)) \times L^2(\Omega)$.

1. Introduction

The paper is devoted to the study of the strongly damped wave equation

$$w_{tt} - \Delta w_t + f(w_t) - \Delta w + g(w) = h. \tag{1.1}$$

The semilinear strongly damped wave equations are quite interesting from a physical viewpoint. For example, they arise in the modeling of the flow of viscoelastic fluids (see [17,7]) as well as in the theory of heat conduction (see [4,10]). One of the most important problems regarding these equations is to analyze their long-time dynamics in terms of attractors. The attractors for such equations have intensively been studied by many authors under different types of hypotheses. We refer to [11,9,25] and the references therein for strongly damped wave equations with the linear damping and subcritical source term. In the critical source term case, the existence of the attractors for strongly damped wave equations with the linear damping was proved in [1] and later in [18]. The regularity of the attractor, established in [1,18], was proved in [19], for the critical source term case. Later in [24], it was shown that the attractor of the strongly damped wave equation with the critical source term, indeed, attracts every bounded subset of $H_0^1(\Omega) \times L^2(\Omega)$ in the norm of $H_0^1(\Omega) \times H_0^1(\Omega)$. In [21], the authors proved the existence and regularity of the uniform attractor for the nonautonomous strongly damped wave equation with the critical source term. The attractors for the strongly damped wave equations with the source term like polynomial of arbitrary degree were investigated in [12]. In the nonlinear subcritical damping term case, the attractors for the strongly damped wave equations

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were studied in [26] and [13]. In [2], the authors investigated the attractors of the abstract second order evolution equation with the damping term depending both on displacement and velocity. In particular, the results obtained in [2] can be applied to the strongly damped wave equation with subcritical nonlinearities. Attractors for strongly damped wave equations with the critical displacement dependent damping and source terms were established in [14]. Recently, in [5] the authors have proved the existence of the attractors for Eq. (1.1), when the source term g is subcritical and nonmonotone damping term f is critical. Later in [6], they have improved this result for the case when both of f and g are critical and $\inf_{x\in R} f'(x) > -\lambda_1$, where λ_1 is the first eigenvalue of Laplace operator.

The goal of this paper is to study the two-dimensional equation (1.1) with exponentially growing damping and source terms, in the space $(H_0^1(\Omega)\cap L^\infty(\Omega))\times L^2(\Omega)$ instead of the usual phase space $H_0^1(\Omega)\times L^2(\Omega)$. Hence, in comparison with the papers mentioned above, we additionally need the $L^\infty(\Omega)$-estimate for the weak solutions. This estimate is also important for the uniqueness and continuous dependence on initial data, in the case when there is no growth condition on the derivative of the source term. To achieve L^∞ regularity of the weak solutions, we reduce the strongly damped wave equation to the heat equation and use the regularity property of the latter (see Lemmas 3.2–3.3).

The paper is organized as follows. In the next section, we state the problem and the main results. In Section 3, we first prove the existence of the weak solution and then establish its L^∞ regularity. After proving L^∞ regularity, we show validity of the inequality (2.6), which implies uniqueness and continuous dependence of the weak solutions on initial data. In Section 4, we first establish the dissipativity, in particular the global boundedness of solutions in $L^\infty(\Omega)$ uniformly with respect to the initial data from a bounded subset of $(H_0^1(\Omega)\cap L^\infty(\Omega))\times L^2(\Omega)$, and then prove asymptotic compactness which together with the existence of the strict Lyapunov function lead to the existence of the global attractor. Finally, in the last section, we give two auxiliary lemmas.

2. Statement of the problem and results

We consider the following initial-boundary value problem:

$$\begin{cases} w_{tt}-\Delta w_t+f(w_t)-\Delta w+g(w)=h(x) & \text{in } (0,\infty)\times\Omega, \\ w=0 & \text{on } (0,\infty)\times\partial\Omega, \\ w(0,\cdot)=w_0, \quad w_t(0,\cdot)=w_1 & \text{in } \Omega, \end{cases} \tag{2.1}$$

where $\Omega\subset R^2$ is a bounded domain with smooth boundary, $h\in L^2(\Omega)$ and the nonlinear functions f, g satisfy the following conditions:

- $$f\in C^1(R), \quad f(0)=0, \quad \inf_{s\in R} f'(s)>-\lambda_1, \tag{2.2}$$
- $$g\in C^1(R), \quad \liminf_{|s|\to\infty}\frac{g(s)}{s}>-\lambda_1, \quad \sup_{s\in R}\big|g(s)\big|e^{-\varepsilon s^2}<\infty, \quad \forall\varepsilon>0, \tag{2.3}$$
- $$\int_{-\infty}^{\infty}\frac{|f'(s)|}{s(f(s)+\lambda_1 s)+1}ds<\infty, \tag{2.4}$$

where $\lambda_1=\inf_{\varphi\in H_0^1(\Omega),\,\varphi\neq 0}\frac{\|\nabla\varphi\|^2_{L^2(\Omega)}}{\|\varphi\|^2_{L^2(\Omega)}}$.

Remark 2.1. The class of functions satisfying (2.2)–(2.4) is quite wide. For example, it is easy to verify that for $\alpha\in[0,1)$ and $\beta\in[0,2)$ the functions $f(s)=se^{|s|^\alpha}$ and $g(s)=s(1+\psi(s))e^{|s|^\beta}+P_n(s)$ satisfy the conditions (2.2)–(2.4), where ψ is a nonnegative, bounded, differentiable function and P_n is a polynomial of degree n.